

Synchronous Machine Modeling



PowerWorld Corporation
2001 S. First St, Suite 203
Champaign, IL 61820
<http://www.powerworld.com>
info@powerworld.com
217 384 6330



PowerWorld
Corporation

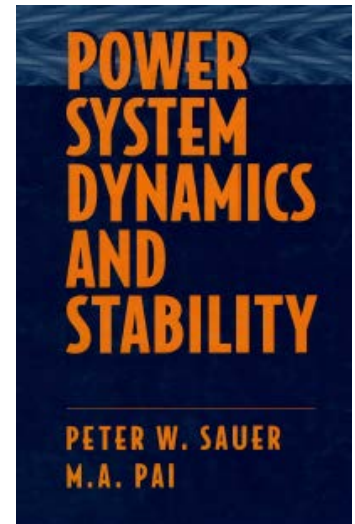
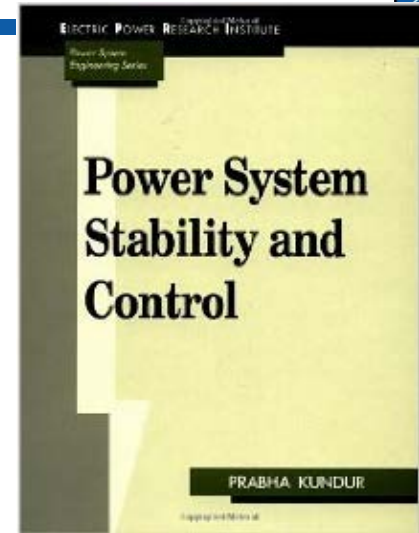
Synchronous Machine Modeling



- Electric machines are used to convert mechanical energy into electrical energy (generators) and from electrical energy into mechanical energy (motors)
 - Many devices can operate in either mode, but are usually customized for one or the other
- Vast majority of electricity is generated using synchronous generators and some is consumed using synchronous motors, so we'll start there
 - Don't worry: We'll talk about wind and solar later!
- There is much literature on subject, and sometimes it is overly confusing with the use of different conventions and nomenclature

Transient Stability Modeling

- A good comprehensive book on this type of analysis is the by Prabha Kundur and is called Power System Stability and Control published in 1994
 - Book is too detailed for a classroom textbook, but it is a really great as a reference book once you're working
- Another good theoretical book is Power System Dynamics and Stability by Peter Sauer and M.A. Pai from 1998.
 - The derivation in this book of the synchronous machine equations very closely matches the equations and nomenclature used in commercial software tools



Synchronous Machine Terminology

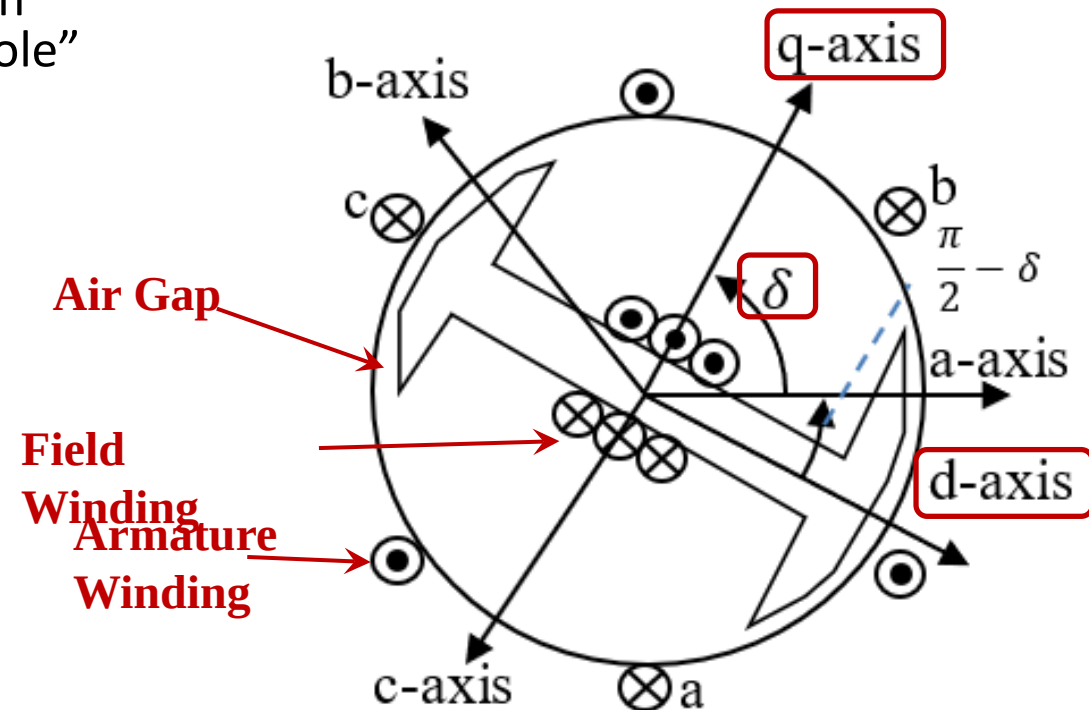


- Positional Winding Terms
 - Stator (Stationary portion of generator)
 - Rotor (Rotating portion of generator)
- Functional Winding Terms
 - Armature Winding (three-phase AC winding that carries the power)
 - Normally this is on the Stator
 - Field Winding (DC current winding)
 - Normally this is on the Rotor
 - Amortisseur Winding (or Damper winding)
 - An extra winding that provides start-up torque and damping
 - Basically a winding that creates a force that attempts to bring machine to synchronous speed (60 Hz)
 - Armature Poles
 - Following slide shows two dots for each (A, B, C) phase → one “in” and one “out”
 - This represents a 2 pole machine
 - If you just repeated this 4 additional times for each phase then you’d have an “8 pole” machine
 - More poles means the machine doesn’t need to spin as fast to create 60 Hz

Modeling the Generator Rotor



- d = Direct Axis
 - Spinning axis directly in line with the “North Pole” of the field winding
- q = Quadrature Axis
 - Spinning axis 90 degrees out of phase with the direct axis
 - (We choose Leading)
- Rotor Angle (δ)
 - Angle between q-axis and Phase A axis
 - (this is arbitrary)

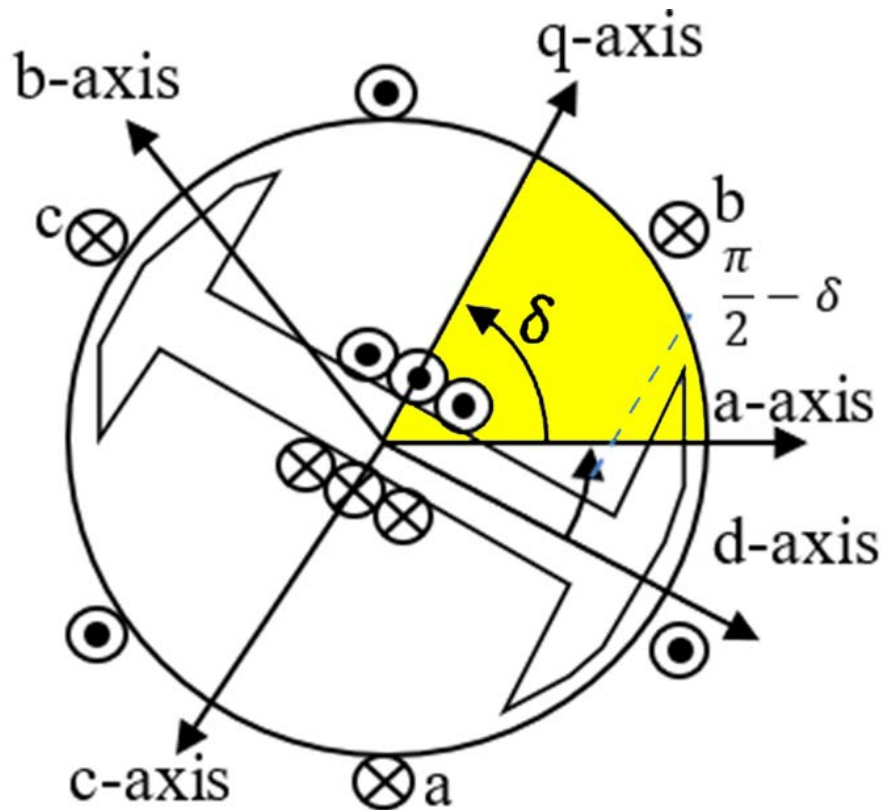


Right-Hand rule defines axes of phase a, b, and c as well as direct axis

Careful of Conventions when looking at Textbooks!

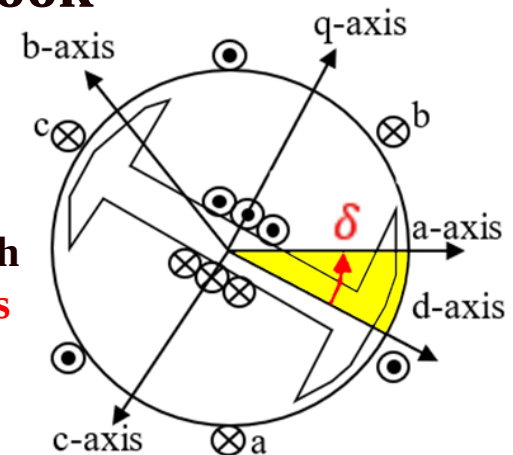
Software Convention Choice

- q-axis leads the d-Axis
- Rotor angle w.r.t. to q-axis
(Also Sauer/Pai book)



Kundar Book

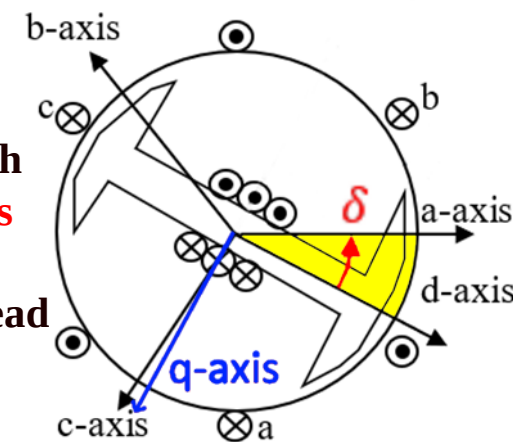
Rotor angle with respect to **d-axis**



Anderson/Fouad Book

Rotor angle with respect to **d-axis**

q-axis **lags** instead of leads

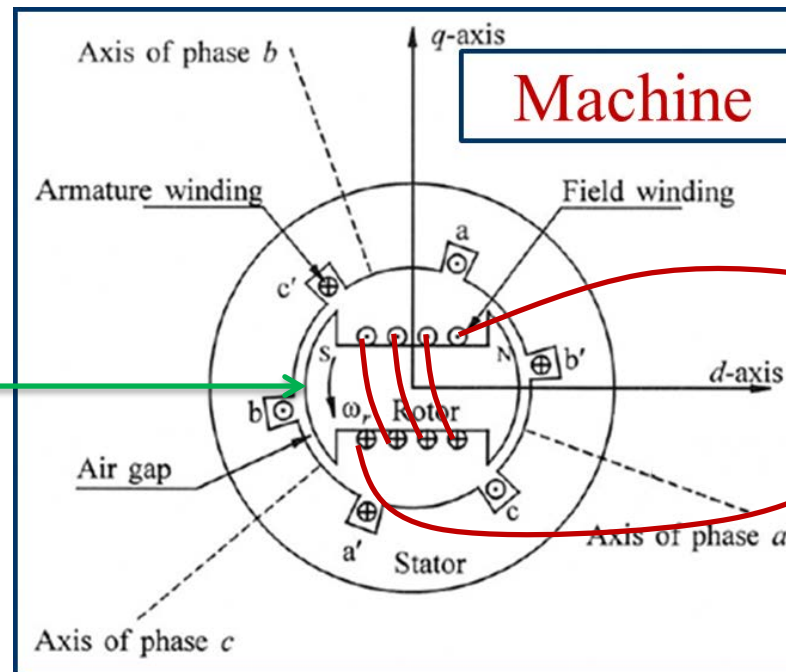


Synchronous Machine



- Basics of Synchronous Machine Model
 - Exciter applies DC current to rotor making it an electromagnet
 - Turbine/Governor spins rotor
 - Spinning magnet creates AC power

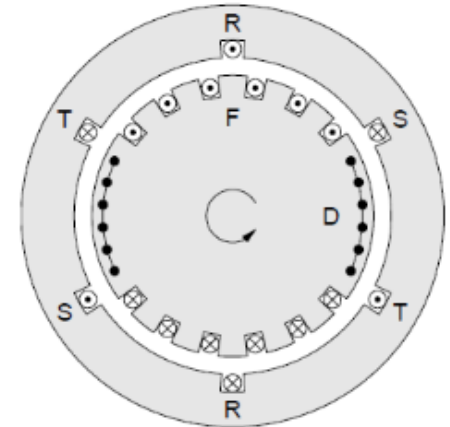
**Turbine/
Governor**
Creates a
mechanical
torque to spin the
rotor



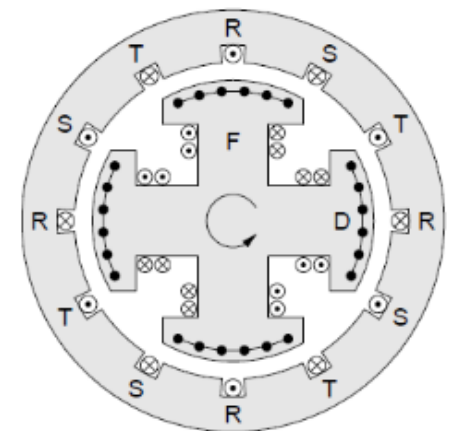
Exciter
Creates a DC
voltage to apply to
Rotor Winding,
resulting in an
electromagnet

Two Main Types of Synchronous Machines

- Round Rotor
 - Air-gap is constant, used with higher speed machines
- Salient Rotor (often called Salient Pole)
 - Air-gap varies circumferentially
 - Used with many pole, slower machines such as hydro
 - Narrowest part of gap in the d-axis and the widest along the q-axis

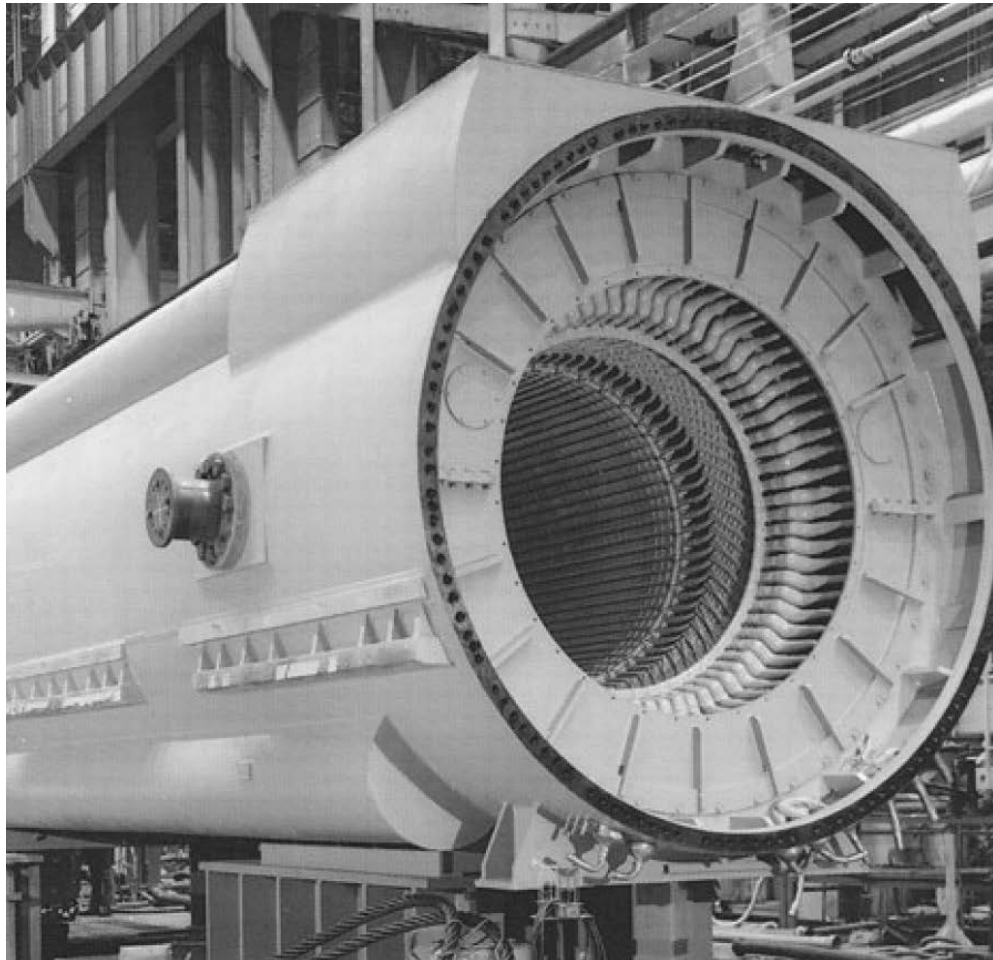


Round rotor
 $p = 2, n = 3000 \text{ min}^{-1}$ for $f = 50 \text{ Hz}$



Salient pole rotor
 $p = 4, n = 1500 \text{ min}^{-1}$ for $f = 50 \text{ Hz}$

Synchronous Machine Stator



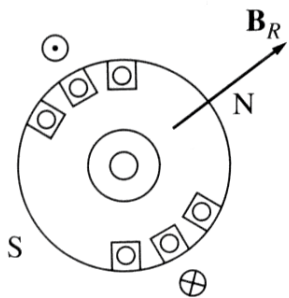
Generator stator showing completed windings for a 757-MVA, 3600-RPM, 60-Hz synchronous generator (Courtesy of General Electric.)

Image Source: Glover/Overbye/Sarma Book, Sixth Edition, Beginning of Chapter 8 Photo

Synchronous Machine Rotors

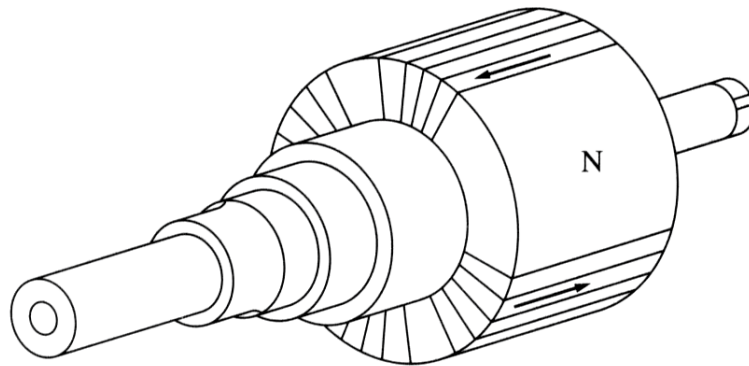


- Rotors are essentially electromagnets

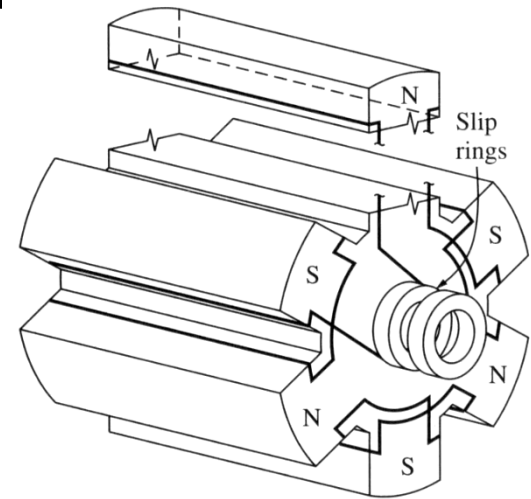


End view

**Two pole (P)
round rotor**



Side view



**Six pole salient
rotor**

Image Source: Dr. Gleb Tcheslavski, ee.lamar.edu/gleb/teaching.htm

Synchronous Machine Rotor



**High pole
salient
rotor**

Shaft

**Part of exciter,
which is used
to control the
field current**

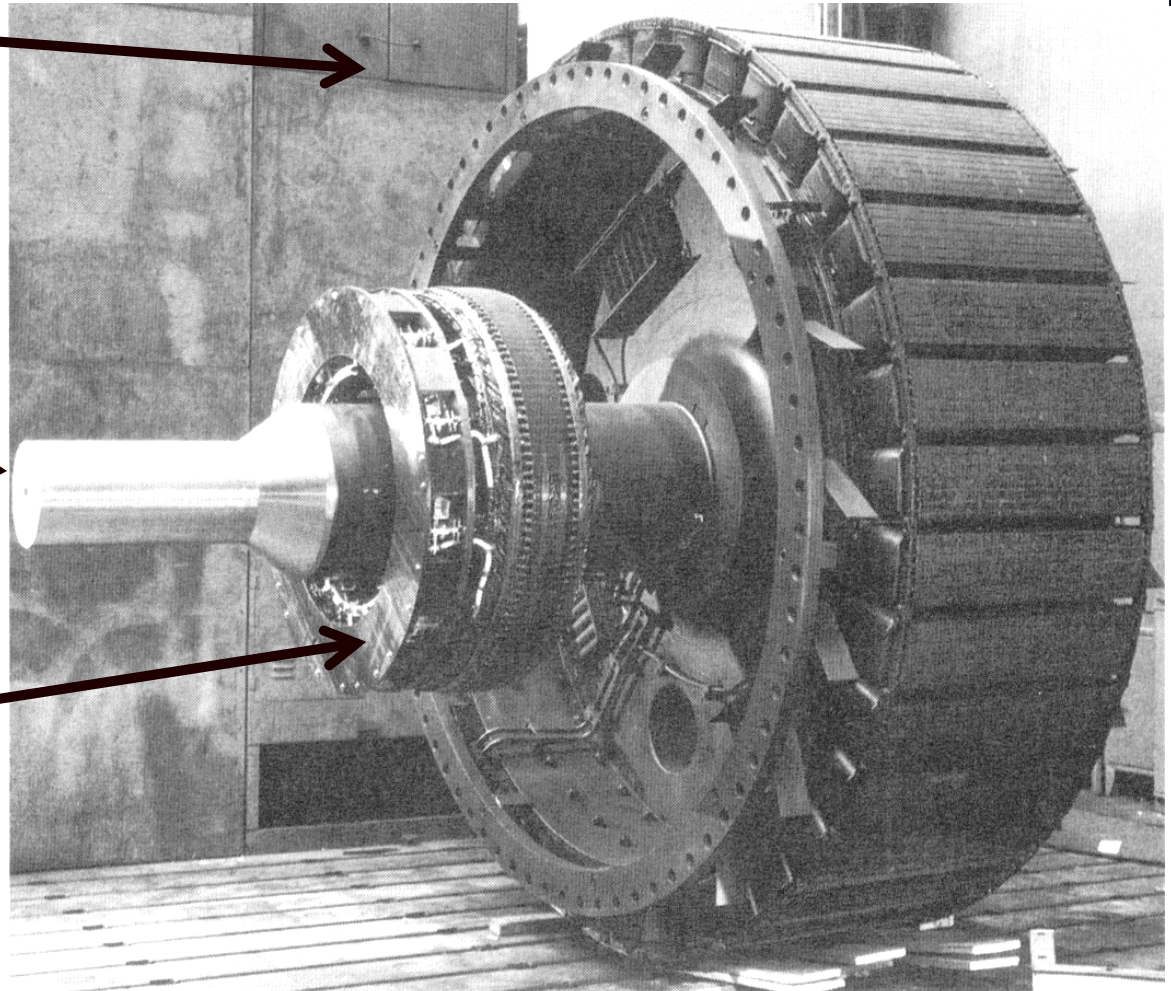


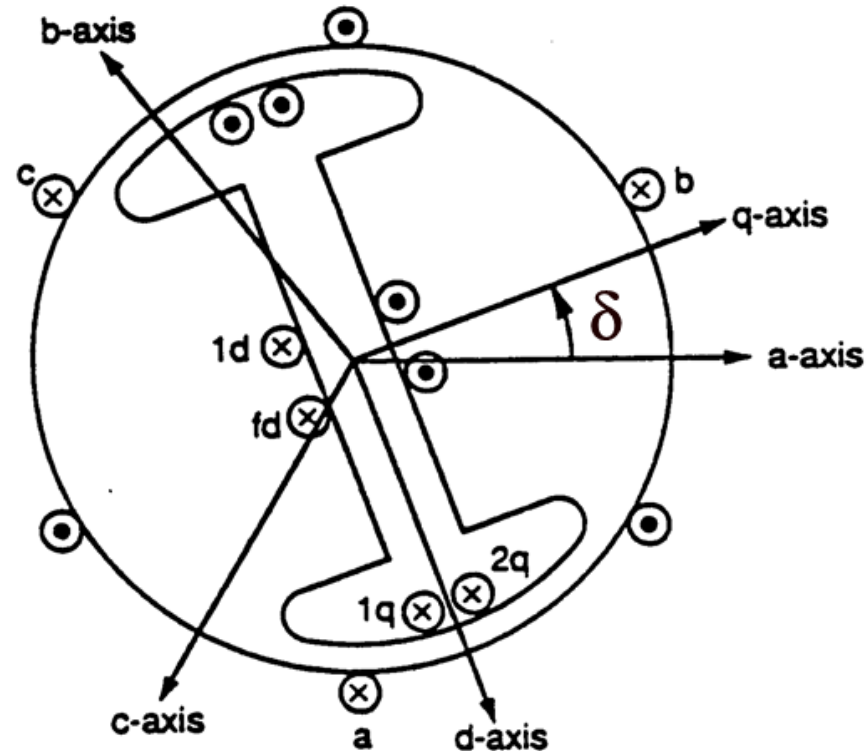
Image Source: Dr. Gleb Tcheslavski, ee.lamar.edu/gleb/teaching.htm

Synchronous Machine Modeling:

Sauer/Pai Book Good Reference



- Stator/Armature Windings
 - 3 ϕ bal. windings (a,b,c)
- Rotor/Field winding
 - Field winding connected to Exciter (fd)
 - “d” axis damper (1d)
 - 2 “q” axis dampers (1q, 2q)
- “Damper” windings do not have external connections
 - They stabilize the electrical operation of the machine
 - Also provide start-up torque for the machine



Dynamic Equations of a Synchronous Machine



- Start with Newton's second law
 - Force = Mass * Acceleration
 - $F = M \frac{dv}{dt}$ and $\frac{dx}{dt} = v$
- We have a rotational system though, so instead we end up with something a little different
 - Torque = Moment of Inertia * Angular Acceleration
 - $T = J \frac{d\omega}{dt}$ and $\frac{d\delta}{dt} = \omega$
- Electrical Equations just apply the same equations to the circuits for the 7 windings a, b, c, fd, 1d, 1q, 2q
 - $v = ir + \frac{d\lambda}{dt}$

Fundamental Laws



- Kirchhoff's Voltage Law, Ohm's Law, Faraday's Law, Newton's Second Law

Stator

$$v_a = i_a r_s + \frac{d\lambda_a}{dt}$$

$$v_b = i_b r_s + \frac{d\lambda_b}{dt}$$

$$v_c = i_c r_s + \frac{d\lambda_c}{dt}$$

Rotor

$$v_{fd} = i_{fd} r_{fd} + \frac{d\lambda_{fd}}{dt}$$

$$v_{1d} = i_{1d} r_{1d} + \frac{d\lambda_{1d}}{dt}$$

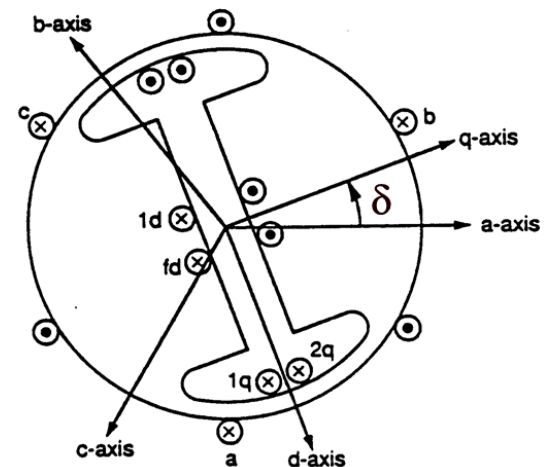
$$v_{1q} = i_{1q} r_{1q} + \frac{d\lambda_{1q}}{dt}$$

$$v_{2q} = i_{2q} r_{2q} + \frac{d\lambda_{2q}}{dt}$$

Shaft

$$\frac{d\theta_{\text{shaft}}}{dt} = \frac{2}{P} \omega$$

$$J \frac{2}{P} \frac{d\omega}{dt} = T_m - T_e - T_f \omega$$



Deriving Equations for Rotor



- Algebra and trigonometry end up being extremely complex
 - Results give inductances between phases that are a function of the cosine of rotor angle. Yuck!
 - Special transformations are done to transform the abc phase quantities into another reference frame
 - Called the dq0 transformation
 - Might hear it called “Park’s Transformation” after Robert H. Park who introduced this concept in 1929 (his was slightly scaled transformation, but was the same idea)
 - Parks’ 1929 paper voted 2nd most important power paper of 20th century (1st was Fortescue’s sym. components)
 - Convention used here is the q-axis leads the d-axis (which is the IEEE standard)

dq0 Transformation



- This is a very similar idea as symmetrical components when discussing fault analysis (different matrix conversion though)
- We can say thanks to engineers who figured this all out for us 80 years ago!
- We end up with 14 equations that go through conversion similar to following
- Also make some more “magic” per unit choices to make things clean-up more

$$\begin{aligned}\psi_a = & -i_a[L_{aa0}+L_{aa2}\cos 2\theta]+i_b[L_{ab0}+L_{aa2}\cos(2\theta+\frac{\pi}{3})] \\ & +i_c[L_{ab0}+L_{aa2}\cos(2\theta-\frac{\pi}{3})]+i_{fd}L_{afd}\cos\theta \\ & +i_{kd}L_{akd}\cos\theta-i_{kq}L_{akq}\sin\theta\end{aligned}$$

$$\begin{aligned}\psi_b = & i_a[L_{ab0}+L_{aa2}\cos(2\theta+\frac{\pi}{3})]-i_b[L_{aa0}+L_{aa2}\cos 2(\theta-\frac{2\pi}{3})] \\ & +i_c[L_{ab0}+L_{aa2}\cos(2\theta-\pi)]+i_{fd}L_{afd}\cos(\theta-\frac{2\pi}{3}) \\ & +i_{kd}L_{akd}\cos(\theta-\frac{2\pi}{3})-i_{kq}L_{akq}\sin(\theta-\frac{2\pi}{3})\end{aligned}$$

$$\begin{aligned}\psi_c = & i_a[L_{ab0}+L_{aa2}\cos(2\theta-\frac{\pi}{3})]+i_b[L_{ab0}+L_{aa2}\cos(2\theta-\pi)] \\ & -i_c[L_{aa0}+L_{aa2}\cos 2(\theta+\frac{2\pi}{3})]+i_{fd}L_{afd}\cos(\theta+\frac{2\pi}{3}) \\ & +i_{kd}L_{akd}\cos(\theta+\frac{2\pi}{3})-i_{kq}L_{akq}\sin(\theta+\frac{2\pi}{3})\end{aligned}$$

dq0

$$\psi_d = -L_d i_d + L_{afd} i_{fd} + L_{akd} i_{kd}$$

$$\psi_q = -L_q i_q + L_{akq} i_{kq}$$

$$\psi_0 = -L_0 i_0$$

Dq0 Transformations



$$\begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} \triangleq T_{dqo} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad \text{or } i, \lambda$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = T_{dqo}^{-1} \begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix}$$

In the next few slides we'll quickly go through how these basic equations are transformed into the standard machine models. The point is to show the physical basis for the models. And there is NO quiz at the end!!

dq0 Transformations



$$T_{dq0} \triangleq \frac{2}{3} \begin{bmatrix} \sin \frac{P}{2} \theta_{shaft} & \sin \left(\frac{P}{2} \theta_{shaft} - \frac{2\pi}{3} \right) & \sin \left(\frac{P}{2} \theta_{shaft} + \frac{2\pi}{3} \right) \\ \cos \frac{P}{2} \theta_{shaft} & \cos \left(\frac{P}{2} \theta_{shaft} - \frac{2\pi}{3} \right) & \cos \left(\frac{P}{2} \theta_{shaft} + \frac{2\pi}{3} \right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

with the inverse,

$$T_{dq0}^{-1} = \begin{bmatrix} \sin \frac{P}{2} \theta_{shaft} & \cos \frac{P}{2} \theta_{shaft} & 1 \\ \sin \left(\frac{P}{2} \theta_{shaft} - \frac{2\pi}{3} \right) & \cos \left(\frac{P}{2} \theta_{shaft} - \frac{2\pi}{3} \right) & 1 \\ \sin \left(\frac{P}{2} \theta_{shaft} + \frac{2\pi}{3} \right) & \cos \left(\frac{P}{2} \theta_{shaft} + \frac{2\pi}{3} \right) & 1 \end{bmatrix}$$

Note that the transformation depends on the shaft angle.

Transformed System



Stator

$$\begin{aligned} v_d &= r_s i_d - \omega \lambda_q + \frac{d\lambda_d}{dt} \\ v_q &= r_s i_q + \omega \lambda_d + \frac{d\lambda_q}{dt} \\ v_o &= r_s i_o + \frac{d\lambda_o}{dt} \end{aligned}$$

Rotor

$$\begin{aligned} v_{fd} &= r_{fd} i_{fd} + \frac{d\lambda_{fd}}{dt} \\ v_{1d} &= r_{1d} i_{1d} + \frac{d\lambda_{1d}}{dt} \\ v_{1q} &= r_{1q} i_{1q} + \frac{d\lambda_{1q}}{dt} \\ v_{2q} &= r_{2q} i_{2q} + \frac{d\lambda_{2q}}{dt} \end{aligned}$$

Shaft

$$\begin{aligned} \frac{d\theta_{shaft}}{dt} &= \frac{2}{P} \omega \\ J \frac{2}{P} \frac{d\omega}{dt} &= T_m - T_e - T_f \omega \end{aligned}$$

Converted from abc reference frame to the dq0 reference frame

Extra Terms show up due to transformation

Electrical & Mechanical Relationships



Electrical system: $v = iR + \frac{d\lambda}{dt}$ (voltage)

$$vi = i^2 R + i \frac{d\lambda}{dt} \quad (\text{power})$$

Mechanical system:

$$J \left(\frac{2}{P} \right) \frac{d\omega}{dt} = T_m - T_e - T_{fw} \quad (\text{torque})$$

P is the number of poles (e.g. 2,4,6)

$$J \left(\frac{2}{P} \right)^2 \omega \frac{d\omega}{dt} = \frac{2}{P} \omega T_m - \frac{2}{P} \omega T_e - \frac{2}{P} \omega T_{fw} \quad (\text{power})$$

**Mechanical
Power**

**Electrical
Torque**

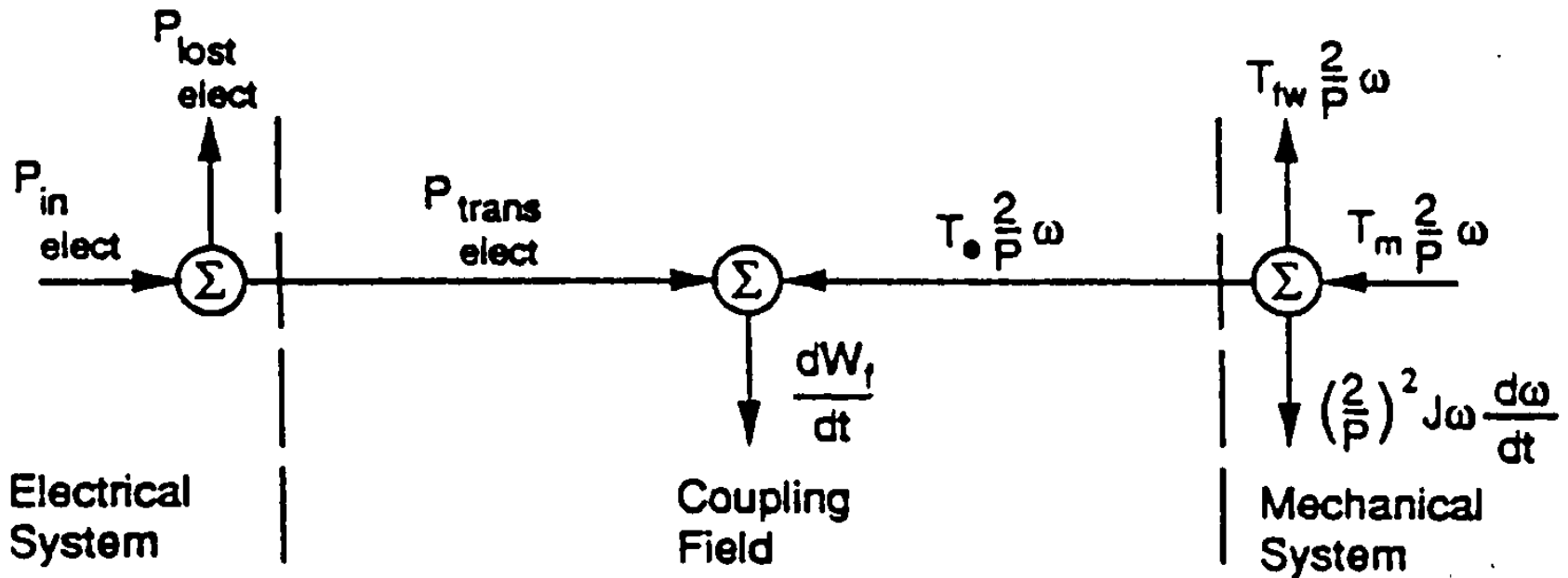
**Friction and
Windage**

Electric Torque Derivation



- Friction and Windage Torque: we ignore
- Mechanical Torque: this is an input to the system
- Electrical Torque
 - Torque is derived by looking at the overall energy balance in the system
 - Three systems: electrical, mechanical and the coupling magnetic field
 - Electrical system losses are in the form of resistance
 - Mechanical system losses are in the form of friction
 - Coupling field is assumed to be lossless, hence we can track how energy moves between the electrical and mechanical systems

Energy Conversion



**No Losses in the
Coupling Field**

Skip the Derivation



- A lot of interesting calculus and algebra using the “Conservative Coupling Field” assumption give us the electrical torque

$$T_{elec} = -\frac{3}{2} \frac{P}{2} (\lambda_d I_q - \lambda_q I_d)$$

- Also we define δ where ω_s is the synchronous speed

$$\delta \triangleq \frac{P}{2} \theta_{shaft} - \omega_s t$$

- This makes our mechanical equations

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$J \frac{2}{p} \frac{d\omega}{dt} = T_m + \left(\frac{3}{2} \right) \left(\frac{P}{2} \right) (\lambda_d i_q - \lambda_q i_d) - T_f \omega$$

Generator Swing Equation



- Anyway, after a lot of additional algebra, software tools model the swing equations as follows with values in per unit
 - $2H \frac{d\omega}{dt} = \frac{P_{mech}}{1+\omega} - T_{elec}$ and $\frac{d\delta}{dt} = \omega$
- If you use a more complete model of the rotor of a generator, then the T_{elec} term has some inherent damping in it
- In academic settings, as we'll introduce in a moment, the rotor modeling has no inherent damping in it (which makes your results really oscillate)
 - To overcome this, folks in the 1960s added an extra $D\omega$ term as follows

$$2H \frac{d\omega}{dt} = \frac{P_{mech} - D\omega}{1 + \omega} - T_{elec}$$

- This term should NOT be used to model the damping in the more accurate rotor models such as GENROU, GENSAL, GENTPF, GENTPJ, etc.
- Friction and Windage term has been removed for our purposes

From here Lots of Per-Unit Choices



- Pages 30 – 42 in the Sauer/Pai textbook go through a ton of per unit scaling and lots of algebra
- Small notation differences, but we end up with the equations on the next page which largely match what Sauer/Pai derive
 - Different Notation though
(Our notation = Sauer/Pai)

$$\bullet \psi'_d = +\psi_{1d}$$

$$\bullet \psi'_q = -\psi_{2q}$$

**Our
Notation**

**Sauer/Pai
Notation**

Variables and Constants



- Per Unit Model Variables
 - ω_s is the synchronous speed ($2\pi 60$)
 - $\Delta\omega_{pu}$ is the deviation of rotor speed away from synchronous speed
 - I_d, I_q, I_0 are the stator current in dq0 reference
 - $V_{dterm}, V_{qterm}, V_{0term}$ are the stator voltage in dq0 reference
 - ψ_d, ψ_q, ψ_0 are the stator flux in the dq0 reference
 - E'_d, E'_q, ψ'_q , and ψ'_d represent per-unitized versions of the rotor fluxes
 - E_{fd} is the field voltage input (from exciter)
 - P_{mech} is the mechanical power input (from turbine/governor)
- Per Unit Model Constants
 - $H, D, R_s, X_d, X'_d, X''_d, X_q, X'_q, X''_q, X_l, T'_{do}, T''_{do}, T'_{qo}, T''_{qo}$

Final Complete Model of Per-Unitized Equations



2 Mechanical Dynamic Equations

$$\frac{d\delta}{dt} = \Delta\omega_{pu} * \omega_s$$

$$2H \frac{d\omega}{dt} = \frac{P_{mech} - D\omega}{1 + \Delta\omega_{pu}} - (\psi_d I_q - \psi_q I_d)$$

3 Stator Dynamic Equations

$$\frac{1}{\omega_s} \frac{d\psi_d}{dt} = R_s I_d + (1 + \Delta\omega_{pu}) \psi_q + V_{dterm}$$

$$\frac{1}{\omega_s} \frac{d\psi_q}{dt} = R_s I_q - (1 + \Delta\omega_{pu}) \psi_d + V_{qterm}$$

$$\frac{1}{\omega_s} \frac{d\psi_0}{dt} = R_s I_0 + V_{0term}$$

Algebraic Relationship between Stator and Rotor Fluxes

$$\psi_d = -I_d X_d'' + E_q' \frac{X_d'' - X_l}{X_d' - X_l} + \psi_q' \frac{X_d' - X_d''}{X_d' - X_l}$$

$$\psi_q = -I_q X_q'' - E_d' \frac{X_q'' - X_l}{X_q' - X_l} - \psi_d' \frac{X_q' - X_q''}{X_q' - X_l}$$

The 3 stator equations define what are known as the stator transients.

4 Rotor Dynamic Equations

$$T_{do}' \frac{dE_q'}{dt} = E_{fd} - E_q' - (X_d - X_d') \left(I_d - \frac{X_d' - X_d''}{(X_d' - X_l)^2} (+\psi_d' + (X_d' - X_l) I_d - E_q') \right)$$

$$T_{qo}' \frac{dE_d'}{dt} = -E_d' + (X_q - X_q') \left(I_q - \frac{X_q' - X_q''}{(X_q' - X_l)^2} (-\psi_q' + (X_q' - X_l) I_q + E_d') \right)$$

$$T_{do}'' \frac{d\psi_d'}{dt} = -\psi_d' - (X_d' - X_l) I_d + E_q'$$

$$T_{qo}'' \frac{d\psi_q'}{dt} = -\psi_q' + (X_q' - X_l) I_q + E_d'$$

Stator Flux Differential Equations



- Stator Differential Equations

$$\frac{1}{\omega_s} \frac{d\psi_d}{dt} = R_s I_d + (1 + \Delta\omega_{pu}) \psi_q + V_{dterm}$$

$$\frac{1}{\omega_s} \frac{d\psi_q}{dt} = R_s I_q - (1 + \Delta\omega_{pu}) \psi_d + V_{qterm}$$

$$\frac{1}{\omega_s} \frac{d\psi_0}{dt} = R_s I_0 + V_{0term}$$

- These have a time constant that is

$$\frac{1}{\omega_s} = \frac{1}{2\pi 60} = 0.00265$$

- About 1/6th of a cycle and is MUCH faster than the phenomena that transient stability is concerned with
- Transient Stability Simulations neglect these

Impact on Studies

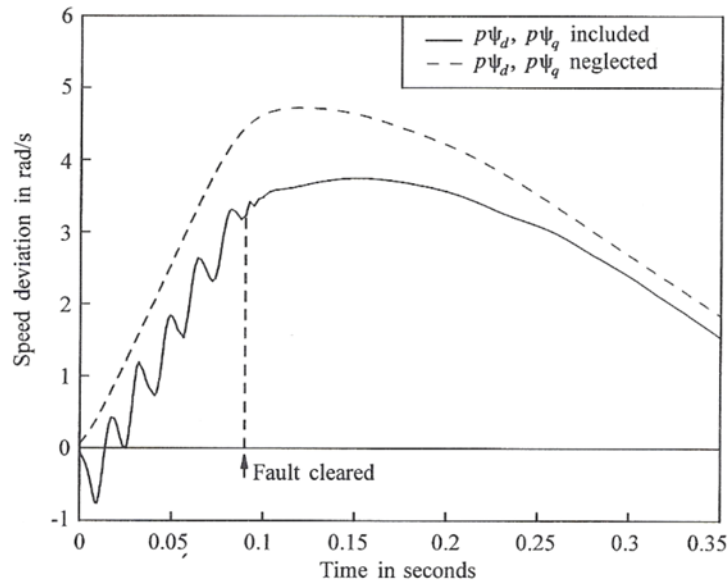


Figure 5.3 Effect of neglecting stator transients on speed deviation

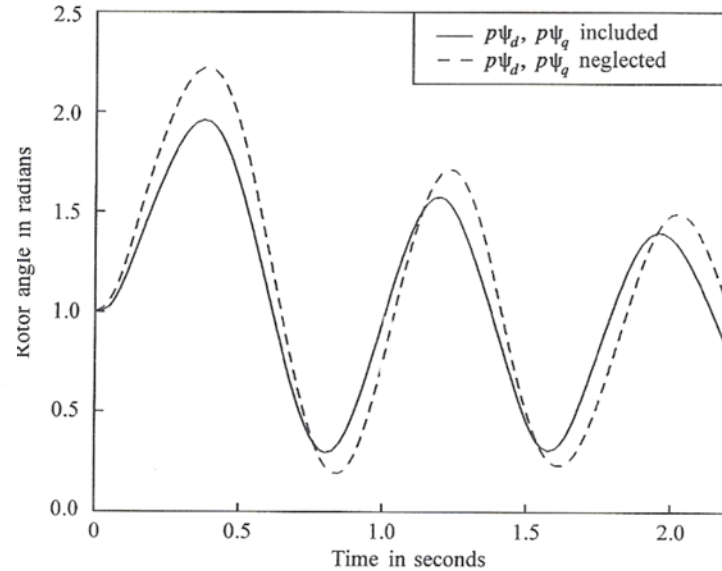


Figure 5.4 Effect of neglecting stator transients on rotor angle swings

Stator transients are not usually considered in transient stability studies

Image Source: P. Kundur, *Power System Stability and Control*, EPRI, McGraw-Hill, 1994

Elimination of Stator Transients



- Mathematics is kind of neat and you get to use terms like “Integral Manifold” and sound really smart
- Easier way to think of it
 - Regardless of what the derivative of these fluxes are, they are multiplied by a very small number $\left(\frac{1}{\omega_s}\right)$ and thus the left-hand side is near zero anyway, so approximation is made

$$\frac{1}{\omega_s} \frac{d\psi_d}{dt} \approx 0 = R_s I_d + (1 + \Delta\omega_{pu}) \psi_q + V_{dterm}$$

$$\frac{1}{\omega_s} \frac{d\psi_q}{dt} \approx 0 = R_s I_q - (1 + \Delta\omega_{pu}) \psi_d + V_{qterm}$$

$$\frac{1}{\omega_s} \frac{d\psi_0}{dt} \approx 0 = R_s I_0 + V_{0term}$$

Stator Equations Becomes Algebraic Network Equation



- This gives us the following

$$V_{dterm} = -R_s I_d - (1 + \Delta\omega_{pu}) \psi_q$$

$$V_{qterm} = -R_s I_q + (1 + \Delta\omega_{pu}) \psi_d$$

Also remember stator flux relationships

$$\psi_d = -I_d X_d'' + E_d' \frac{X_d'' - X_l}{X_d' - X_l} + \psi_d' \frac{X_d' - X_d''}{X_d' - X_l}$$

$$\psi_q = -I_q X_q'' - E_d' \frac{X_q'' - X_l}{X_q' - X_l} - \psi_q' \frac{X_q' - X_q''}{X_q' - X_l}$$

- Take the algebraic flux relationship and plug it in

$$V_{dterm} = -R_s I_d - (1 + \Delta\omega_{pu}) \left(-I_q X_q'' - E_d' \frac{X_q'' - X_l}{X_q' - X_l} - \psi_q' \frac{X_q' - X_q''}{X_q' - X_l} \right)$$

$$V_{qterm} = -R_s I_q + (1 + \Delta\omega_{pu}) \left(-I_d X_d'' + E_d' \frac{X_d'' - X_l}{X_d' - X_l} + \psi_d' \frac{X_d' - X_d''}{X_d' - X_l} \right)$$

- Group Terms

$$V_{dterm} = -R_s I_d + (1 + \Delta\omega_{pu}) X_q'' I_q + (1 + \Delta\omega_{pu}) \left(+E_d' \frac{X_q'' - X_l}{X_q' - X_l} + \psi_q' \frac{X_q' - X_q''}{X_q' - X_l} \right)$$

$$V_{qterm} = -R_s I_q - (1 + \Delta\omega_{pu}) X_d'' I_d + (1 + \Delta\omega_{pu}) \left(+E_d' \frac{X_d'' - X_l}{X_d' - X_l} + \psi_d' \frac{X_d' - X_d''}{X_d' - X_l} \right)$$

Stator Equation Becomes Network Equation



- Now define

$$E_d'' = +E_d' \frac{X_q'' - X_l}{X_q' - X_l} + \psi_q' \frac{X_q' - X_q''}{X_q' - X_l}$$

$$E_q'' = +E_q' \frac{X_d'' - X_l}{X_d' - X_l} + \psi_d' \frac{X_d' - X_d''}{X_d' - X_l}$$

- This gives us the following

$$V_{dterm} = -R_s I_d + (1 + \Delta\omega_{pu}) X_d'' I_q + E_d'' (1 + \Delta\omega_{pu})$$

$$V_{qterm} = -R_s I_q - (1 + \Delta\omega_{pu}) X_d'' I_d + E_q'' (1 + \Delta\omega_{pu})$$

- Consider term $(1 + \Delta\omega_{pu})$ that remains
 - Similar to multiplying all the transmission line X values by per unit frequency to scale the network impedances as system frequency changes
 - No one does this in transient stability, so we just remove the purple
 - We leave the scaling of the internal voltage though

$$V_{dterm} = E_d'' (1 + \Delta\omega_{pu}) - R_s I_d + X_q'' I_q$$

$$V_{qterm} = E_q'' (1 + \Delta\omega_{pu}) - R_s I_q - X_d'' I_d$$

Final Complete Model of Per-Unitized Equations



Algebraic Relationships

$$E_d'' = +E_d' \frac{X_q'' - X_l}{X_q' - X_l} + \psi_q' \frac{X_q' - X_q''}{X_q' - X_l}$$

$$E_q'' = +E_q' \frac{X_d'' - X_l}{X_d' - X_l} + \psi_d' \frac{X_d' - X_d''}{X_d' - X_l}$$

$$\psi_d = -I_d X_d'' + E_q''$$

$$\psi_q = -I_q X_q'' - E_d''$$

$$V_{dterm} = E_d'' (1 + \Delta\omega_{pu}) - R_s I_d + X_q'' I_q$$

$$V_{qterm} = E_q'' (1 + \Delta\omega_{pu}) - R_s I_q - X_d'' I_d$$

6 Differential Equations

$$1. \quad \frac{d\delta}{dt} = \Delta\omega_{pu} * \omega_s$$

$$2. \quad 2H \frac{d\omega}{dt} = \frac{P_{mech} - D\omega}{1 + \Delta\omega_{pu}} - (\psi_d I_q - \psi_q I_d)$$

$$3. \quad T_{do}' \frac{dE_q'}{dt} = E_{fd} - E_q' - (X_d - X_d') \left(I_d - \frac{X_d' - X_d''}{(X_d' - X_l)^2} (+\psi_d' + (X_d' - X_l) I_d - E_q') \right)$$

$$4. \quad T_{qo}' \frac{dE_d'}{dt} = -E_d' + (X_q - X_q') \left(I_q - \frac{X_q' - X_q''}{(X_q' - X_l)^2} (-\psi_q' + (X_q' - X_l) I_q + E_d') \right)$$

$$5. \quad T_{do}'' \frac{d\psi_d'}{dt} = -\psi_d' - (X_d' - X_l) I_d + E_q'$$

$$6. \quad T_{qo}'' \frac{d\psi_q'}{dt} = -\psi_q' + (X_q' - X_l) I_q + E_d'$$

Field Voltage and Current



- Field Voltage (E_{fd}) is an input from the exciter
- The field current, I_{fd} , is defined in steady-state as

$$I_{fd} = E_{fd} / X_{md}$$

- Traditional, when we talk about a “field current” we are actually mean the product of the field current and the mutual inductance $I_{fd}X_{md}$ (also written $I_{fd}L_{ad}$)

$$L_{ad}I_{fd} = E'_q + (X_d - X'_d) \left(I_d - \frac{X'_d - X''_d}{(X'_d - X_l)^2} (+\psi'_d + (X'_d - X_l)I_d - E'_q) \right)$$

$$T'_{do} \frac{dE'_q}{dt} = E_{fd} - L_{ad}I_{fd}$$

Multiple Machines



- This math is all very beautiful, but it is all related to a single machine that is connected to a three-phase system
- We need a similar conversion for the abc quantities at all the network buses
- This requires us to do some more beautiful math
- We will first convert all network abc quantities to a common reference frame we call the Synchronous Reference Frame

Synchronously Rotating Reference Frame



- That conversion is chosen as follows
 - It was chosen because the math works out so nicely

$$T_{sync} = \frac{2}{3} \begin{bmatrix} \cos(\omega t) & \cos\left(\omega t - \frac{2\pi}{3}\right) & \cos\left(\omega t + \frac{2\pi}{3}\right) \\ -\sin(\omega t) & -\sin\left(\omega t - \frac{2\pi}{3}\right) & -\sin\left(\omega t + \frac{2\pi}{3}\right) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \Rightarrow \begin{bmatrix} v_D \\ v_Q \\ v_O \end{bmatrix} = T_{sync} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

- Assume you have a 3-phase balanced voltage

$$\begin{bmatrix} v_D \\ v_Q \\ v_O \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\omega t) & \cos\left(\omega t - \frac{2\pi}{3}\right) & \cos\left(\omega t + \frac{2\pi}{3}\right) \\ -\sin(\omega t) & -\sin\left(\omega t - \frac{2\pi}{3}\right) & -\sin\left(\omega t + \frac{2\pi}{3}\right) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} V_t \cos(\omega t + \alpha) \\ V_t \cos\left(\omega t + \alpha - \frac{2\pi}{3}\right) \\ V_t \cos\left(\omega t + \alpha + \frac{2\pi}{3}\right) \end{bmatrix}$$

Synchronous Reference Frame for 3-Phase Balanced Operation



- Now multiply these out

$$\begin{aligned}v_d &= V_t \frac{2}{3} \left[+\cos(\omega t) \cos(\omega t + \alpha) + \cos\left(\omega t - \frac{2\pi}{3}\right) \cos\left(\omega t + \alpha - \frac{2\pi}{3}\right) + \cos\left(\omega t + \frac{2\pi}{3}\right) \cos\left(\omega t + \alpha + \frac{2\pi}{3}\right) \right] \\v_q &= V_t \frac{2}{3} \left[-\sin(\omega t) \cos(\omega t + \alpha) - \sin\left(\omega t - \frac{2\pi}{3}\right) \cos\left(\omega t + \alpha - \frac{2\pi}{3}\right) - \sin\left(\omega t + \frac{2\pi}{3}\right) \cos\left(\omega t + \alpha + \frac{2\pi}{3}\right) \right] \\v_0 &= V_t \frac{2}{3} \left[\frac{1}{2} \cos(\omega t + \alpha) + \frac{1}{2} \cos\left(\omega t + \alpha - \frac{2\pi}{3}\right) + \frac{1}{2} \cos\left(\omega t + \alpha + \frac{2\pi}{3}\right) \right]\end{aligned}$$

- The v_0 value will evaluation to 0.0
 - Three cosine waves shifted by $\frac{2\pi}{3}$ sum to 0.0
- Then use of the following trigonometry

$$\begin{aligned}\cos\left(x + \frac{2\pi}{3}\right) \cos\left(y + \frac{2\pi}{3}\right) &= +\frac{3}{4} \sin(x) \sin(y) + \frac{1}{4} \cos(x) \cos(y) + \frac{\sqrt{3}}{4} \sin(x) \cos(y) + \frac{\sqrt{3}}{4} \cos(x) \sin(y) \\ \cos\left(x - \frac{2\pi}{3}\right) \cos\left(y - \frac{2\pi}{3}\right) &= +\frac{3}{4} \sin(x) \sin(y) + \frac{1}{4} \cos(x) \cos(y) - \frac{\sqrt{3}}{4} \sin(x) \cos(y) - \frac{\sqrt{3}}{4} \cos(x) \sin(y) \\ \sin\left(x + \frac{2\pi}{3}\right) \cos\left(y + \frac{2\pi}{3}\right) &= +\frac{\sqrt{3}}{4} \sin(x) \sin(y) - \frac{\sqrt{3}}{4} \cos(x) \cos(y) + \frac{1}{4} \sin(x) \cos(y) - \frac{3}{4} \cos(x) \sin(y) \\ \sin\left(x - \frac{2\pi}{3}\right) \cos\left(y - \frac{2\pi}{3}\right) &= -\frac{\sqrt{3}}{4} \sin(x) \sin(y) + \frac{\sqrt{3}}{4} \cos(x) \cos(y) + \frac{1}{4} \sin(x) \cos(y) - \frac{3}{4} \cos(x) \sin(y)\end{aligned}$$

Synchronous Reference Frame for 3-Phase Balanced Operation



- Lots of terms cancel and you end up with

$$\begin{aligned}v_d &= V_t \frac{2}{3} \left[+\frac{3}{2} \cos(\omega t) \cos(\omega t + \alpha) + \frac{3}{2} \sin(\omega t) \sin(\omega t + \alpha) \right] \\v_q &= V_t \frac{2}{3} \left[-\frac{3}{2} \sin(\omega t) \cos(\omega t + \alpha) + \frac{3}{2} \cos(\omega t) \sin(\omega t + \alpha) \right] \\v_0 &= 0\end{aligned}$$

- The 2/3 and 3/2 terms cancel out and you get

$$\begin{aligned}v_d &= V_t [+ \cos(\omega t) \cos(\omega t + \alpha) + \sin(\omega t) \sin(\omega t + \alpha)] \\v_q &= V_t [- \sin(\omega t) \cos(\omega t + \alpha) + \cos(\omega t) \sin(\omega t + \alpha)] \\v_0 &= 0\end{aligned}$$

- More Trigonometry and it simplifies to

$$\begin{aligned}v_d &= V_t \cos(\alpha) \\v_q &= V_t \sin(\alpha) \\v_0 &= 0\end{aligned}$$

**All the (ωt) terms
cancel out!**

Synchronous Reference Frame for 3-Phase Balanced Operation



- A-phase sinusoidal is $V_t \cos(\omega t + \alpha)$
 - Magnitude of V_t and phase difference from the “synchronous reference frame” of α
- Value converted to the synchronous reference frame are
$$\begin{bmatrix} v_D \\ v_Q \\ v_O \end{bmatrix} = \begin{bmatrix} +V_t \cos(\alpha) \\ +V_t \sin(\alpha) \\ 0 \end{bmatrix}$$
- That means we can treat this like a complex number and just jump right to
$$v_D + jv_Q = V_t \cos(\alpha) + jV_t \sin(\alpha) = V_t e^{j\alpha}$$
- This is exactly what we always do in steady state power flow equations, so this is nothing new!

Must Apply Same Synchronous Reference Frame to Machine



- Remember that the abc quantities are converted to the machine reference frame.
- We then need to additionally transform those to the Network Reference

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = T_{dqoi}^{-1} \begin{bmatrix} V_{di} \\ V_{qi} \\ V_{oi} \end{bmatrix} \Rightarrow \begin{bmatrix} v_D \\ v_Q \\ v_O \end{bmatrix} = T_{sync} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = T_{sync} T_{dqoi}^{-1} \begin{bmatrix} V_{di} \\ V_{qi} \\ V_{oi} \end{bmatrix}$$

- Need the matrix in yellow for direct conversion
- “Synchronous Reference Frame”
→ Just call it the “Network Reference”

- Network Reference Frame
- Machine Reference Frame

$$\begin{bmatrix} v_D \\ v_Q \\ v_O \end{bmatrix} = T_{sync} T_{dqoi}^{-1} \begin{bmatrix} V_{di} \\ V_{qi} \\ V_{oi} \end{bmatrix}$$

Converting Machine to Network Reference Frame



- Remember how we converted the abc phase stator quantities to the dq0 reference frame

$$T_{dq0}^{-1} = \begin{bmatrix} \sin \frac{P}{2} \theta_{shaft} & \cos \frac{P}{2} \theta_{shaft} & 1 \\ \sin \left(\frac{P}{2} \theta_{shaft} - \frac{2\pi}{3} \right) & \cos \left(\frac{P}{2} \theta_{shaft} - \frac{2\pi}{3} \right) & 1 \\ \sin \left(\frac{P}{2} \theta_{shaft} + \frac{2\pi}{3} \right) & \cos \left(\frac{P}{2} \theta_{shaft} + \frac{2\pi}{3} \right) & 1 \end{bmatrix}$$

- Also remember our choice of

$$\delta \triangleq \frac{P}{2} \theta_{shaft} - \omega_s t$$

- Thus $\frac{P}{2} \theta_{shaft} = \omega_s t + \delta$

Converting Machine to Network Reference Frame



- This makes our transformation matrix for a particular machine “i” the following

$$T_{dq0i}^{-1} = \begin{bmatrix} \sin(\omega t + \delta) & \cos(\omega t + \delta) & 1 \\ \sin\left(\omega t + \delta - \frac{2\pi}{3}\right) & \cos\left(\omega t + \delta - \frac{2\pi}{3}\right) & 1 \\ \sin\left(\omega t + \delta + \frac{2\pi}{3}\right) & \cos\left(\omega t + \delta + \frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

- This means for our machine reference frame to network reference frame we need the following

$$T_{sync} T_{dq0}^{-1} = \frac{2}{3} \begin{bmatrix} \cos(\omega t) & \cos\left(\omega t - \frac{2\pi}{3}\right) & \cos\left(\omega t + \frac{2\pi}{3}\right) \\ -\sin(\omega t) & -\sin\left(\omega t - \frac{2\pi}{3}\right) & -\sin\left(\omega t + \frac{2\pi}{3}\right) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} \sin(\omega t + \delta) & \cos(\omega t + \delta) & 1 \\ \sin\left(\omega t + \delta - \frac{2\pi}{3}\right) & \cos\left(\omega t + \delta - \frac{2\pi}{3}\right) & 1 \\ \sin\left(\omega t + \delta + \frac{2\pi}{3}\right) & \cos\left(\omega t + \delta + \frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

Converting Machine to Network Reference Frame



$$T_{sync}T_{dq0}^{-1} = \frac{2}{3} \begin{bmatrix} \cos(\omega t) & \cos\left(\omega t - \frac{2\pi}{3}\right) & \cos\left(\omega t + \frac{2\pi}{3}\right) \\ -\sin(\omega t) & -\sin\left(\omega t - \frac{2\pi}{3}\right) & -\sin\left(\omega t + \frac{2\pi}{3}\right) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} \sin(\omega t + \delta) & \cos(\omega t + \delta) & 1 \\ \sin\left(\omega t + \delta - \frac{2\pi}{3}\right) & \cos\left(\omega t + \delta - \frac{2\pi}{3}\right) & 1 \\ \sin\left(\omega t + \delta + \frac{2\pi}{3}\right) & \cos\left(\omega t + \delta + \frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

- Use the trigonometry identities
 - $-\sin(x) \sin(x + \delta) = -\frac{1}{2}\cos(\delta) + \frac{1}{2}\cos(2x + \delta)$
 - $-\sin(x) \cos(x + \delta) = +\frac{1}{2}\sin(\delta) - \frac{1}{2}\sin(2x + \delta)$
 - $+\cos(x) \cos(x + \delta) = +\frac{1}{2}\cos(\delta) + \frac{1}{2}\cos(2x + \delta)$
 - $+\cos(x) \sin(x + \delta) = +\frac{1}{2}\sin(\delta) + \frac{1}{2}\sin(2x + \delta)$
- When you group terms all the $\cos(2x + \delta)$ and $\sin(2x + \delta)$ terms are going to cancel out and you only have the

$$T_{sync}T_{dq0}^{-1} = \frac{2}{3} \begin{bmatrix} +\frac{3}{2}\sin(\delta) & +\frac{3}{2}\cos(\delta) & 0 \\ -\frac{3}{2}\cos(\delta) & +\frac{3}{2}\sin(\delta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

All the (ωt) terms cancel out!

Converting Machine to Network Reference Frame



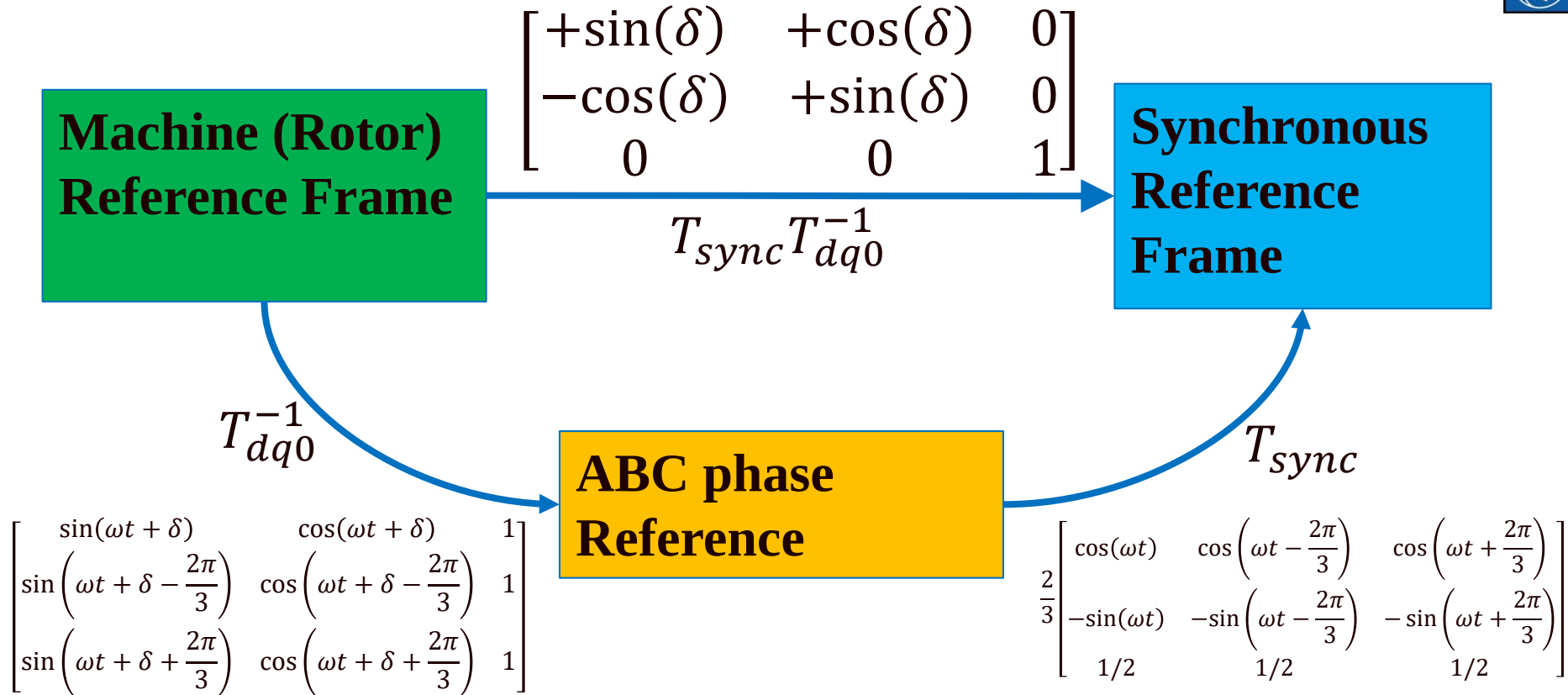
- Thus the conversion from Machine $\rightarrow$ Network is

$$T_{sync}T_{dq0}^{-1} = \begin{bmatrix} +\sin(\delta) & +\cos(\delta) & 0 \\ -\cos(\delta) & +\sin(\delta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

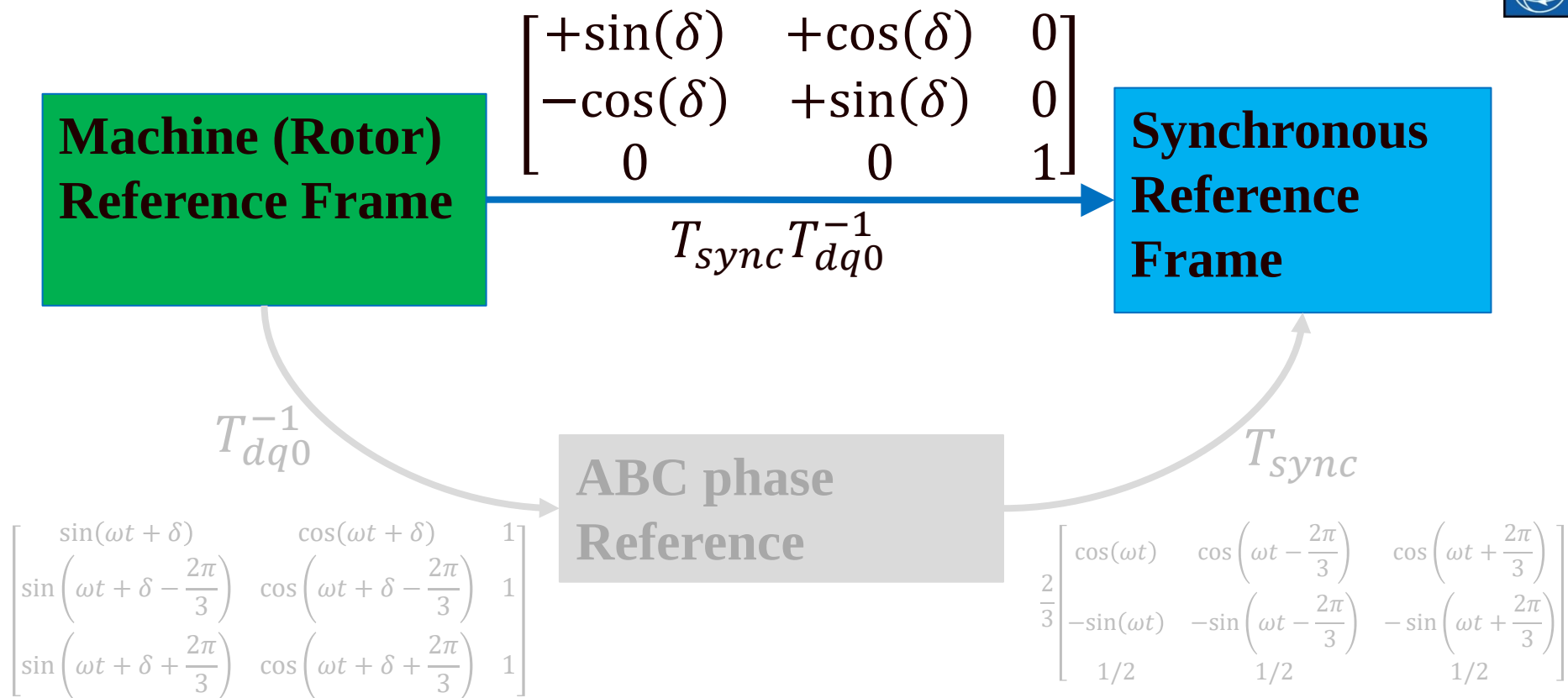
- And the conversion from Network $\rightarrow$ Machine is

$$T_{dq0}T_{sync}^{-1} = \begin{bmatrix} +\sin(\delta) & -\cos(\delta) & 0 \\ +\cos(\delta) & +\sin(\delta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Various Reference Conversions



We shall never speak of the ABC Phase Reference Again!



We're done with considering the “abc phase” Reference. Good to know it's there though if we ever want to output to EMTP tool

Machine to Network Reference Frame Conversion



- For our final result we will omit the “zero” values
 - We are assuming 3-phase balanced operation so these are not going to come into play anyway

$$\begin{bmatrix} V_{dnetwork} \\ V_{qnetwork} \end{bmatrix} = \begin{bmatrix} +\sin(\delta) & +\cos(\delta) \\ -\cos(\delta) & +\sin(\delta) \end{bmatrix} \begin{bmatrix} V_{dmachine} \\ V_{qmachine} \end{bmatrix}$$

$$\begin{bmatrix} V_{dmachine} \\ V_{qmachine} \end{bmatrix} = \begin{bmatrix} +\sin(\delta) & -\cos(\delta) \\ +\cos(\delta) & +\sin(\delta) \end{bmatrix} \begin{bmatrix} V_{dnetwork} \\ V_{qnetwork} \end{bmatrix}$$

- You can also treat the d/q values as real and imaginary numbers and the conversion is then simply complex number rotation

$$(V_{dnetwork} + jV_{qnetwork}) = (V_{dmachine} + jV_{qmachine})e^{+j(\delta - \frac{\pi}{2})}$$

$$(V_{dmachine} + jV_{qmachine}) = (V_{dnetwork} + jV_{qnetwork})e^{-j(\delta - \frac{\pi}{2})}$$

Final Complete Model of Per-Unitized Dynamic Equations



Algebraic Relationships

$$E_d'' = +E_d' \frac{X_q'' - X_l}{X_q' - X_l} + \psi_q' \frac{X_q' - X_q''}{X_q' - X_l}$$

$$E_q'' = +E_q' \frac{X_d'' - X_l}{X_d' - X_l} + \psi_d' \frac{X_d' - X_d''}{X_d' - X_l}$$

$$\psi_d = -I_d X_d'' + E_q''$$

$$\psi_q = -I_q X_q'' - E_d''$$

$$V_{dterm} = E_d'' (1 + \Delta\omega_{pu}) - R_s I_d + X_q'' I_q$$

$$V_{qterm} = E_q'' (1 + \Delta\omega_{pu}) - R_s I_q - X_d'' I_d$$

6 Differential Equations

$$\frac{d\delta}{dt} = \Delta\omega_{pu} * \omega_s$$

$$2H \frac{d\omega}{dt} = \frac{P_{mech} - D\omega}{1 + \Delta\omega_{pu}} - (\psi_d I_q - \psi_q I_d)$$

$$T_{do}' \frac{dE_q'}{dt} = E_{fd} - E_q' - (X_d - X_d') \left(I_d - \frac{X_d' - X_d''}{(X_d' - X_l)^2} (+\psi_d' + (X_d' - X_l) I_d - E_q') \right)$$

$$T_{qo}' \frac{dE_d'}{dt} = -E_d' + (X_q - X_q') \left(I_q - \frac{X_q' - X_q''}{(X_q' - X_l)^2} (-\psi_q' + (X_q' - X_l) I_q + E_d') \right)$$

$$T_{do}'' \frac{d\psi_d'}{dt} = -\psi_d' - (X_d' - X_l) I_d + E_q'$$

$$T_{qo}'' \frac{d\psi_q'}{dt} = -\psi_q' + (X_q' - X_l) I_q + E_d'$$

Network Equations Interface Simplification



- Equations that model the connection of the generator to the network are

$$V_{dterm} = E_d'' (1 + \Delta\omega_{pu}) - R_s I_d + X_q'' I_q$$

$$V_{qterm} = E_q'' (1 + \Delta\omega_{pu}) - R_s I_q - X_d'' I_d$$

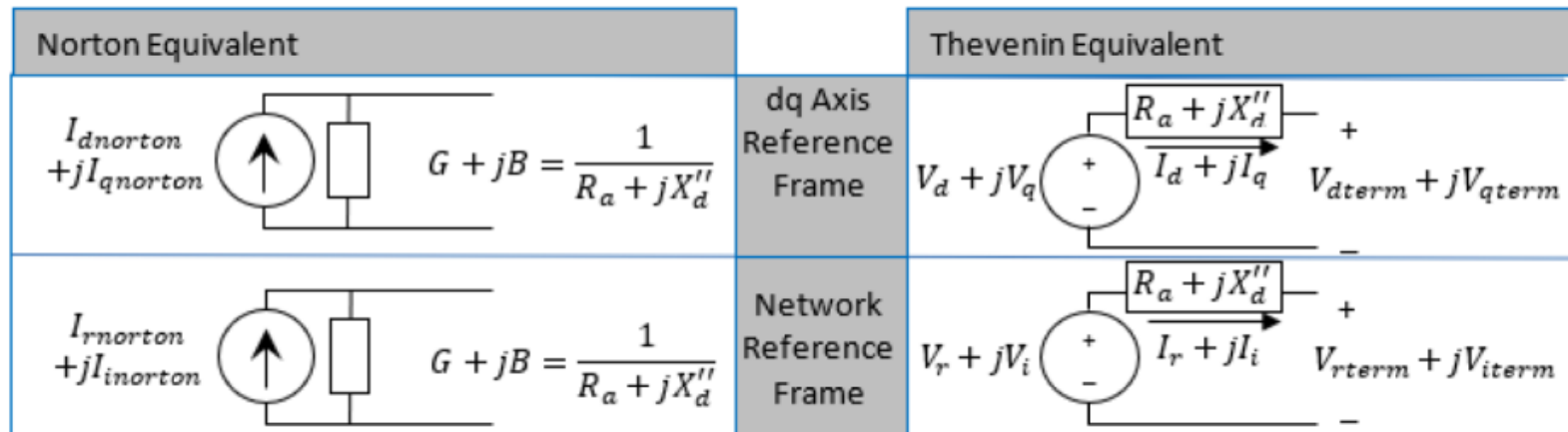
- For GENROU and GENSAL models, two simplifying assumptions are made
 - $X_q'' = X_d''$ (there is no “subtransient saliency”), this gives us a simple circuit equation
 - The value of X_d'' used in the network boundary equations DOES NOT saturate (this makes the machine internal impedance in network equations constant)
- This becomes a simple circuit equation

$$V_{dterm} + jV_{qterm} = (1 + \Delta\omega_{pu})(E_d'' + jE_q'') - (R_s + jX_d'')(I_d + jI_q)$$

Network Equations Interface Simplification



- Remember we need to convert between the “dq” (machine) reference and the Network Reference
 - $(V_r + jV_i) = (1 + \Delta\omega_{pu})(E_d'' + jE_q'')e^{+j(\delta - \frac{\pi}{2})}$
 - $(I_r + jI_i) = (I_d + jI_q)e^{+j(\delta - \frac{\pi}{2})}$
 - $V_{rterm} + jV_{iterm} = (V_r + jV_i) - (R_s + jX_d'')(I_r + jI_i)$
- This is a simple circuit equation we can model as Norton Equivalent / Thevenin Equivalent



An Aside Preparing for Renewable Models Later



- The synchronous machine models all build differential equations that result in a voltage source behind an impedance
- When moving over the network boundary equations this is a very good thing!
- This is a much more numerically robust arrangement
 - Current is limited by the internal impedance and voltage
 - Voltage is limited by the internal voltage
- This is NOT going to be the case with the renewable models like REGC_A, or with particular load models such as LD1PAC and DER_A

An Aside about Simplifying Assumptions for GENROU/GENSAL



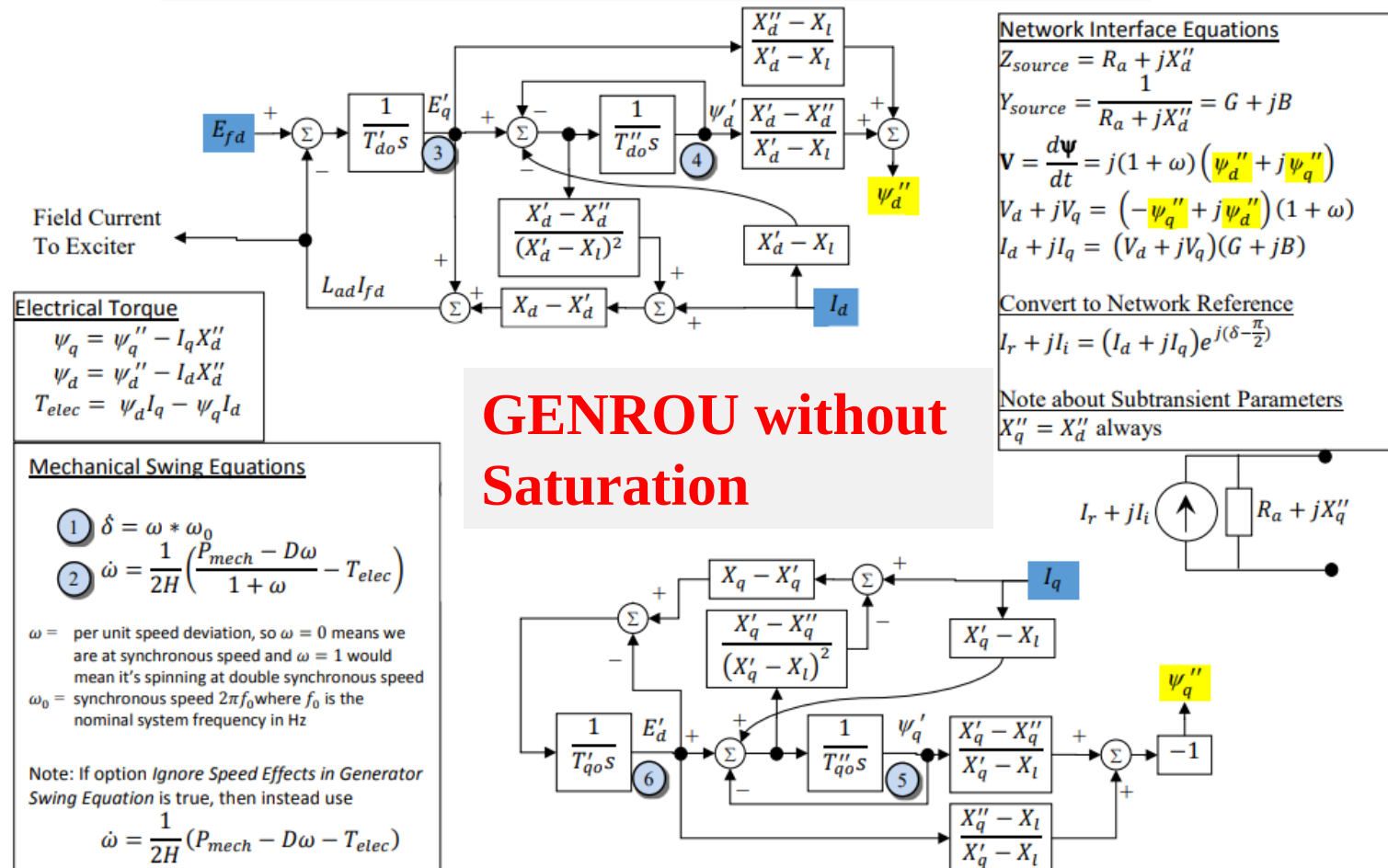
- Models GENTPF and GENTPJ do not make either assumption on the previous pages
 - They allow sub-transient saliency ($X_q'' \neq X_d''$ allowed)
 - Also, both of those terms may also saturate
- This make the network interface no longer a simple circuit equation and more complicated
 - This makes the software harder to write and was an impediment to doing this in 1970, but no reason we need to make that assumption now
- New models we'll mention later do not make these assumptions either
 - GENQEC

Summary of Equations as Block Diagram



Machine Model GENROU

Machine Model GENROU
Solid Rotor Generator represented by equal mutual inductance rotor modeling



Summary of What will be Modeled in Transient Stability



- Differential Equations will be modeled directly
 - The output of the differential equations will include the internal voltage to apply to the network and the internal impedance
 - $(1 + \Delta\omega_{pu})(E_d'' + jE_q'')$
 - $(R_s + jX_d'')$
- Algebraic network equations are calculated across the entire system
 - Output of this will be new stator currents that are inputs to the synchronous machine model
 - Output of new terminal voltages, currents, and power will all feedback to other dynamic models (exciter, governor, etc) as well

Reminder!



- Be careful looking at other textbooks and academic papers
- We have made arbitrary choices!
 - Our choice of defining of the rotor angle as the angle between the a-axis and the q-axis effected all this math
 - The definition of q-axis leading or lagging the d-axis effected all this math too
- I've wasted many days in the past decade getting confused when reading books and papers!
 - Books and Papers may have various 90 degree phase shifts and sign differences
 - All comes down to these arbitrary choices

Pause for a Moment



- We just got to a point where all this math has cleverly removed all the (ωt) terms from everything!
- We have also converted the translation between the network equations and the rotating generator into a simple complex number rotation
- That was a very large amount of math
- The choice of these reference frame transformations matrices took engineers decades to figure out!
- Pause to thank the engineers from 50 - 80 years ago for working through all this

Presentation so far applies to all Synchronous Machine Models

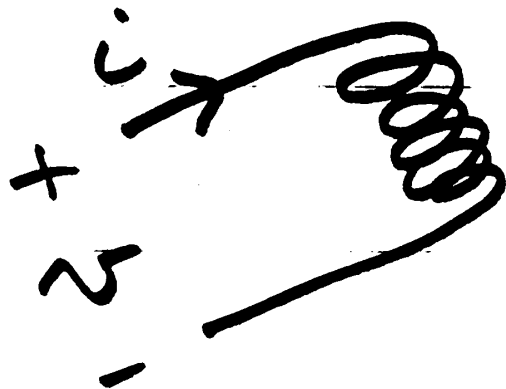


- However,
 - Magnetic Saturation hasn't been introduced yet
 - Simplification due to machine construction for Salient Pole Machines has been ignored
- All synchronous machine models build from this point
 - GENROU/GENROE (most directly, but with additive saturation)
 - GENSAL/GENSAE (some older treatment of saturation, simplify Q axis)
 - GENTPF (some simplification to dynamic model plus multiplicative saturation)
 - GENTPJ (same as GENTPF but stator current also effects saturation)
 - Prospective new models: GENQEC

Nonlinear Magnetic Circuits



- Nonlinear magnetic models are needed because magnetic materials will saturate
 - Saturation means increasingly large amounts of current are needed to increase the flux density

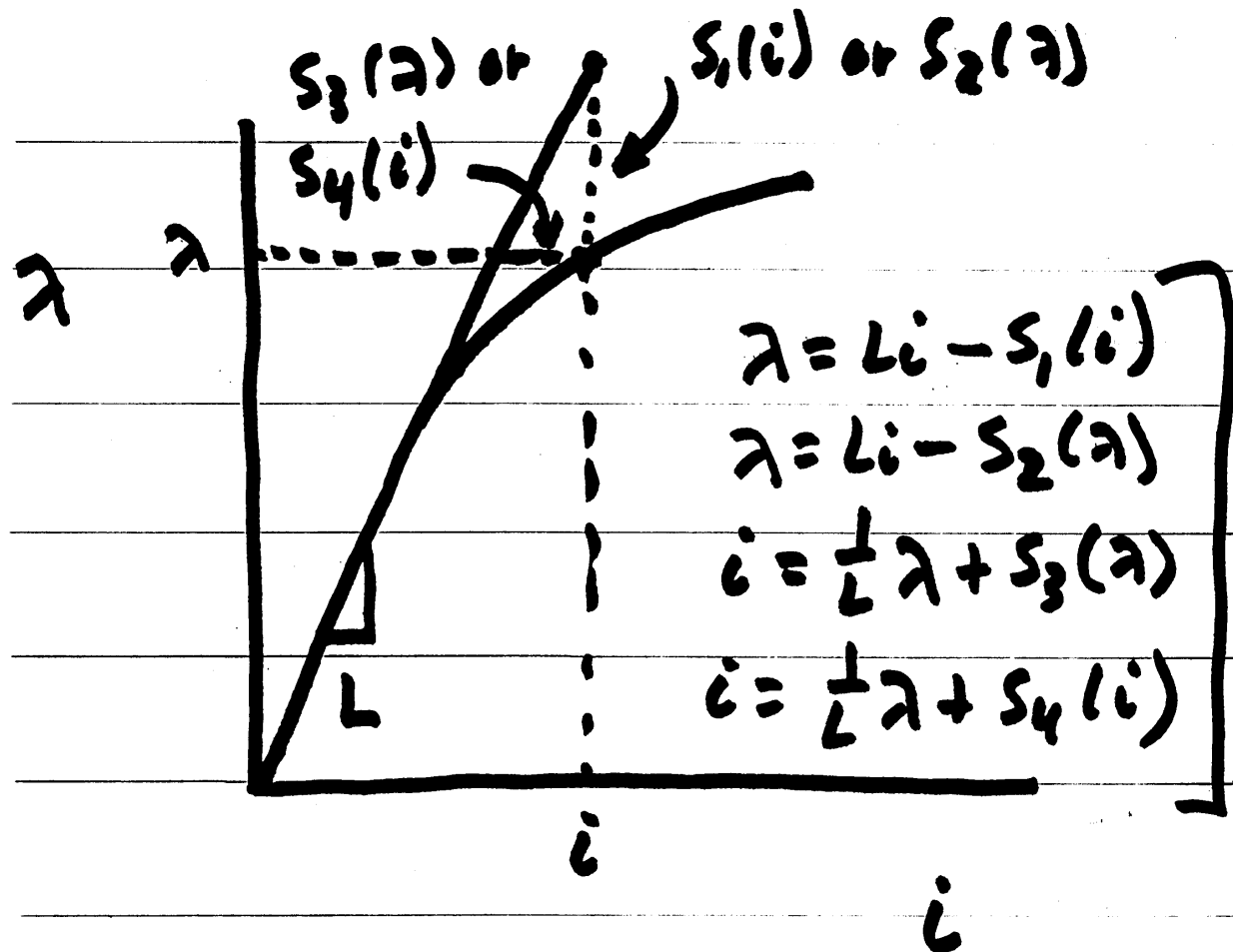


$$R = 0$$

$$v = \frac{d\lambda}{dt} = N \frac{d\phi}{dt}$$

When linear $\lambda = Li$

Saturation



Relative Magnetic Strength Levels



- Earth's magnetic field is between 30 and 70 mT (0.3 to 0.7 gauss)
- A refrigerator magnet might have 0.005 T
- A commercial neodymium magnet might be 1 T
- A magnetic resonance imaging (MRI) machine would be between 1 and 3 T
- Strong lab magnets can be 10 T
- Frogs can be levitated at 16 T (see www.ru.nl/hfml/research/levitation/diamagnetic)
- A neutron star can have 1 to 100 MT!

Magnetic Saturation and Hysteresis



- The below image shows the saturation curves for various materials

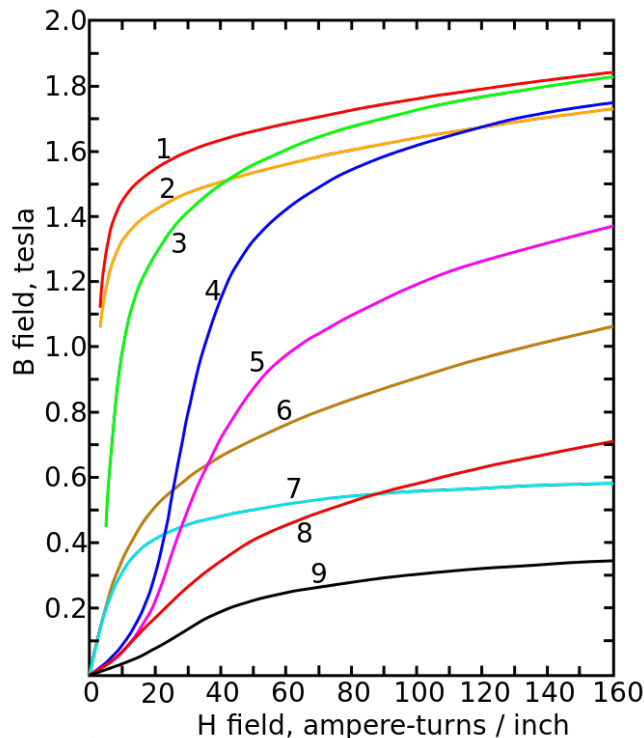


Image Source:
[en.wikipedia.org/wiki/Saturation_\(magnetic\)](https://en.wikipedia.org/wiki/Saturation_(magnetic))

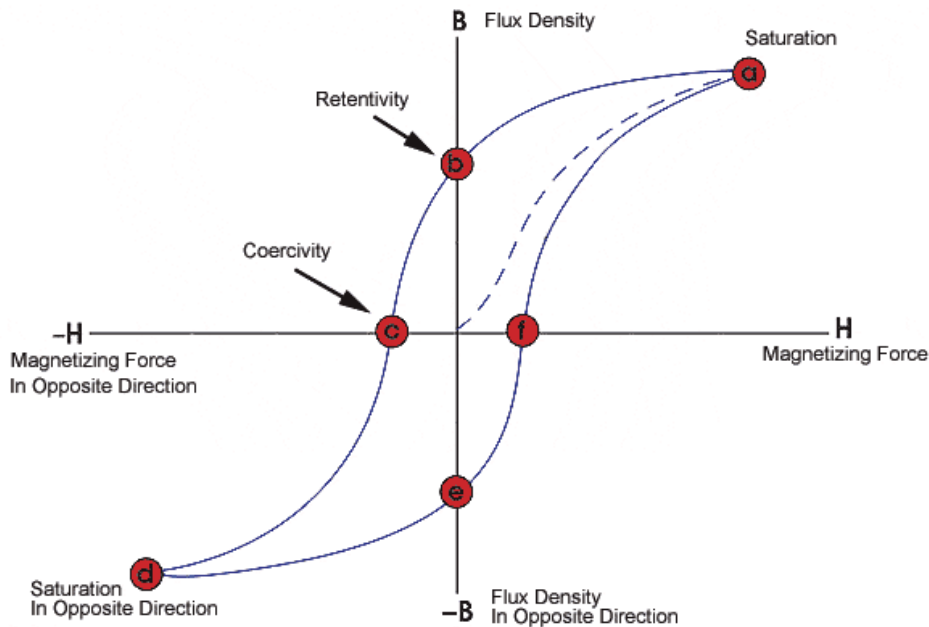
Magnetization curves of 9 ferromagnetic materials, showing saturation. 1. Sheet steel, 2. Silicon steel, 3. Cast steel, 4. Tungsten steel, 5. Magnet steel, 6. Cast iron, 7. Nickel, 8. Cobalt, 9. Magnetite; highest saturation materials can get to around 2.2 or 2.3T

H is proportional to current

Magnetic Saturation and Hysteresis



- Magnetic materials also exhibit hysteresis, so there is some residual magnetism when the current goes to zero; design goal is to reduce the area enclosed by the hysteresis loop



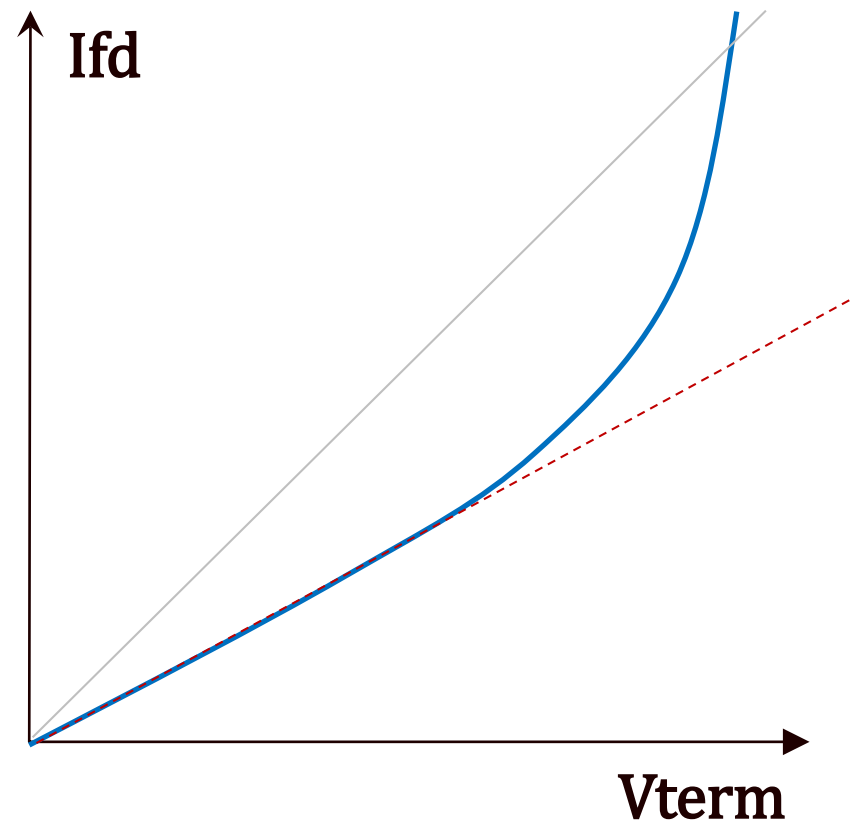
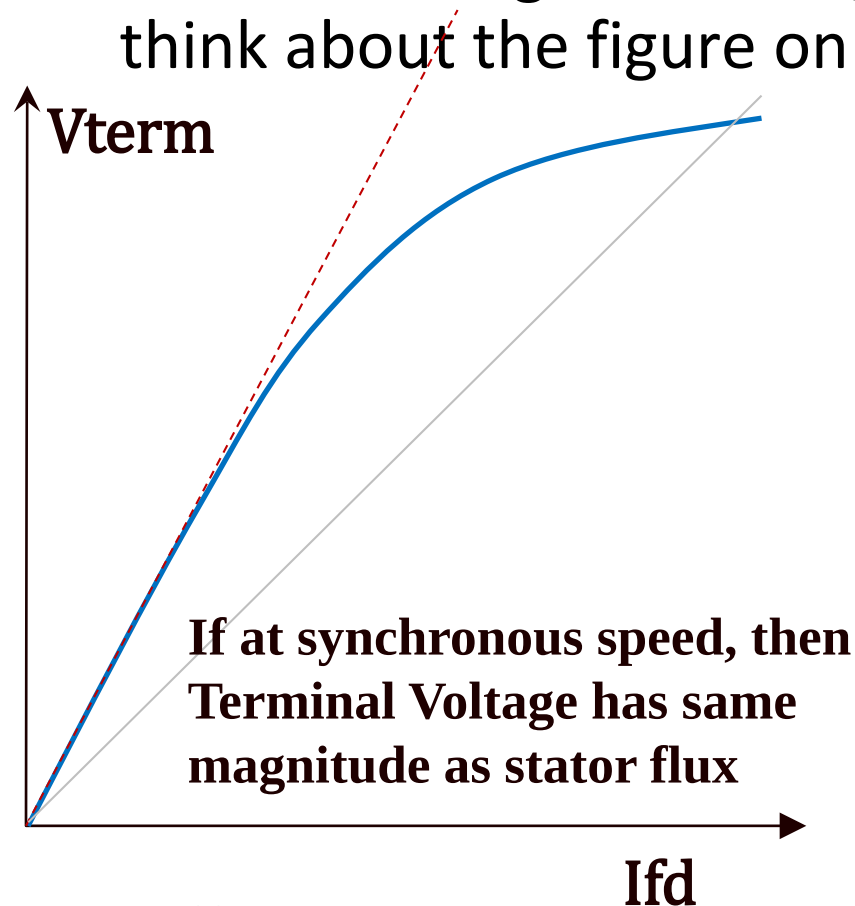
To minimize the amount of magnetic material, and hence cost and weight, electric machines are designed to operate close to saturation

Typical Saturation Tests Plot

Terminal Voltage vs. E_{fd}



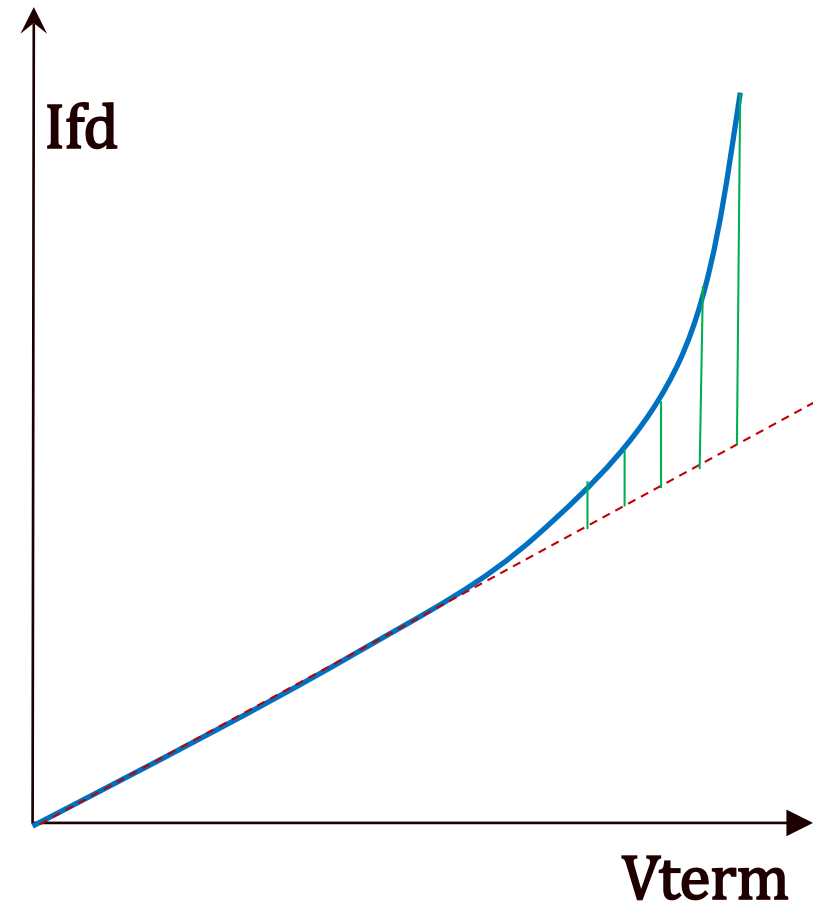
- For those doing generator testing they will do tests to build the figure on left, but I like to flip axes and think about the figure on the right instead



What is showing the Saturation?



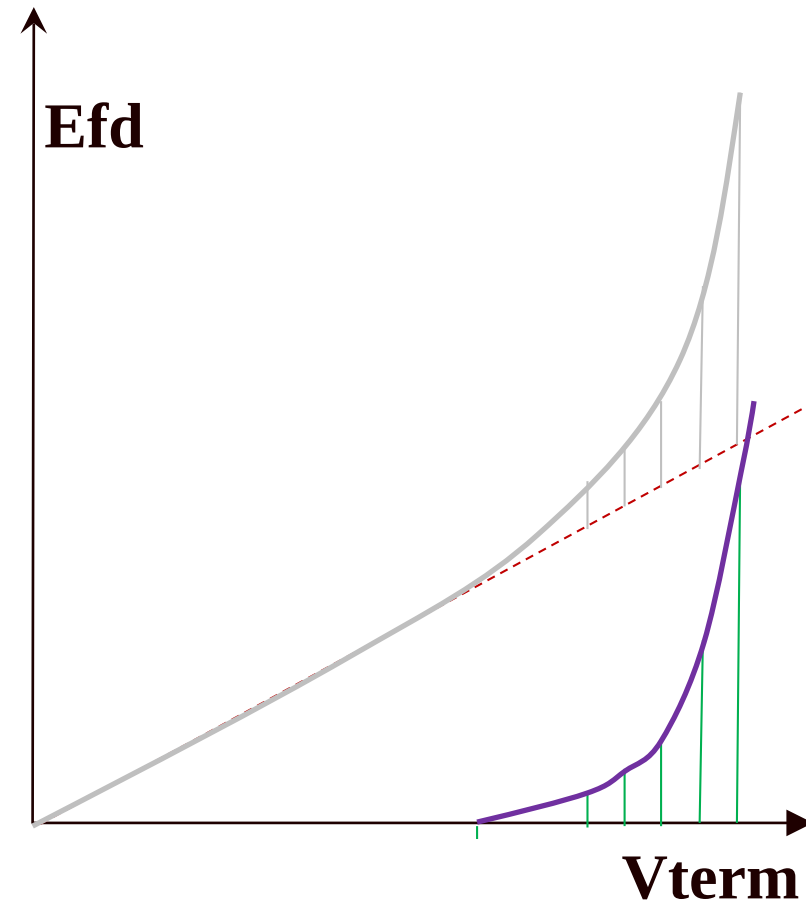
- Saturation represents how extra field current is necessary to obtain a particular terminal voltage
 - It doesn't follow the red line which would represent a linear magnetic relationship
- The Saturation Function will be a function of Flux (Voltage) and represents the green segments



Saturation is the “Extra” above what linear term represents



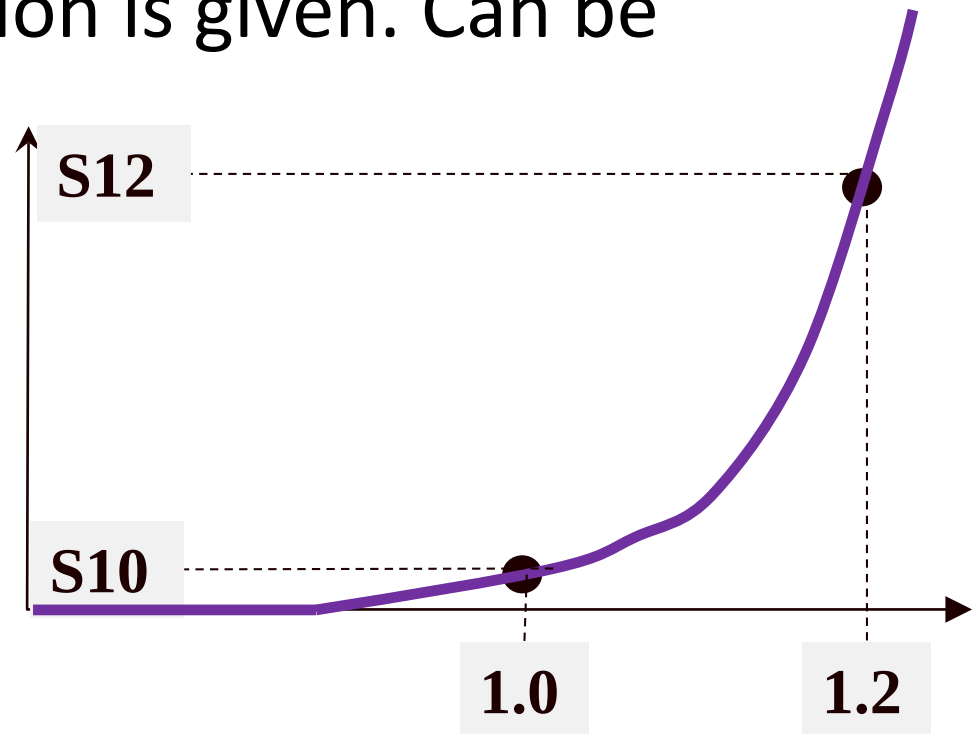
- Saturation Function is determined from generator testing by getting the purple line at the right
- Any function that approximates the purple shape can be used
- Different software tools and particular models use different functions



Description of Saturation in Software



- Two points on the saturation curve are provided as input data
- The saturation function is given. Can be different types
 - Quadratic
 - Scaled Quadratic
 - Exponential



Saturation Function Types



Name	Function	Which Platform
Quadratic	$Sat(x) = B(x - A)^2$	GE PSLF PowerWorld Simulator option
Scaled Quadratic	$Sat(x) = \frac{B(x - A)^2}{x}$	PTI PSS/E, PowerWorld Simulator Option
Exponential	$Sat(x) = Bx^A$	BPA IPF Specific models in PTI PSS/E Specific models in PowerWorld Simulator

Scaled Quadratic Function

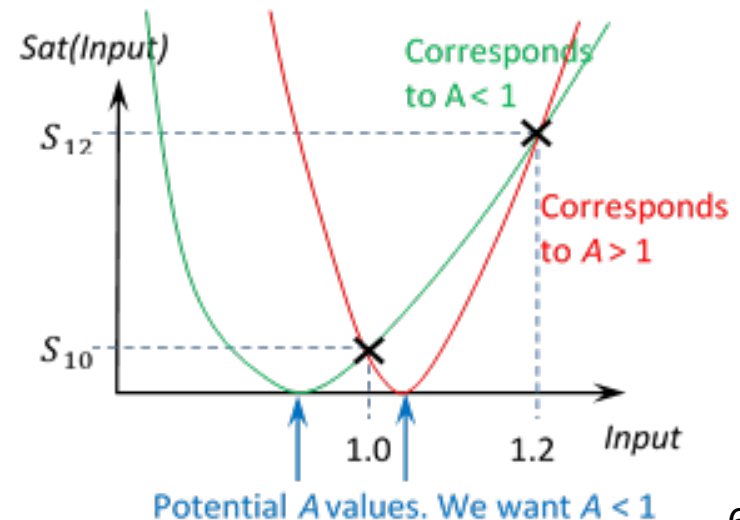


- $$Sat(x) = \begin{cases} \frac{B(x-A)^2}{x} & \text{If } x > A \\ 0 & \text{If } x \leq A \end{cases}$$

- A and B coefficients are calculated from the two equations defined by the two given points
- There are two solutions though: we take the one representing the Green Curve ($A < 1$)

$$S_{10} = \frac{B(1.0 - A)^2}{1.0} \quad S_{12} = \frac{B(1.2 - A)^2}{1.2}$$

Solve one for B and substitute into the second. **Groups terms and you get quadratic function with two solutions. Use the solution for which $A < 1$.**



$$\left(\frac{S_{12}}{S_{10}} - 1.2\right) + \left(-2\frac{S_{12}}{S_{10}} + 2\right)A + \left(\frac{S_{12}}{S_{10}} - \frac{1.0}{1.2}\right)A^2 = 0$$

Quadratic Function



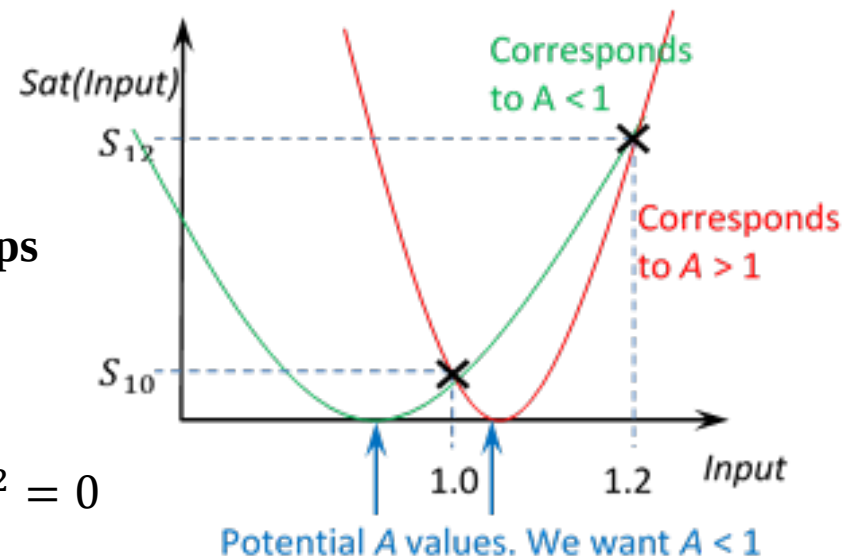
- $Sat(x) = \begin{cases} B(x - A)^2 & \text{If } x > A \\ 0 & \text{If } x \leq A \end{cases}$
 - A and B coefficients are calculated from the two equations defined by the two given points
 - There are two solutions though: we take the one representing the Green Curve ($A < 1$)

$$S_{10} = B(1.0 - A)^2$$

$$S_{12} = B(1.2 - A)^2$$

Solve one for B and substitute into the second. **Groups terms and you get quadratic function with two solutions. Use the solution for which $A < 1$.**

$$\left(\frac{S_{12}}{S_{10}} - 1.44\right) + \left(-2\frac{S_{12}}{S_{10}} + 2.4\right)A + \left(\frac{S_{12}}{S_{10}} - 1.0\right)A^2 = 0$$



Exponential Function



- $Sat(x) = B(x)^A$
 - A and B coefficients are calculated from the two equations defined by the two given points

$$S_{10} = B(1.0)^A$$

$$S_{10} = B$$

$$S_{12} = B(1.2)^A$$

$$S_{12} = S_{10}(1.2)^A$$

$$\frac{S_{12}}{S_{10}} = (1.2)^A$$

$$\ln\left(\frac{S_{12}}{S_{10}}\right) = \ln[(1.2)^A] = A \ln(1.2)$$

$$B = S_{10}$$

$$A = \frac{\ln\left(\frac{S_{12}}{S_{10}}\right)}{\ln(1.2)}$$

Implementing Saturation Models



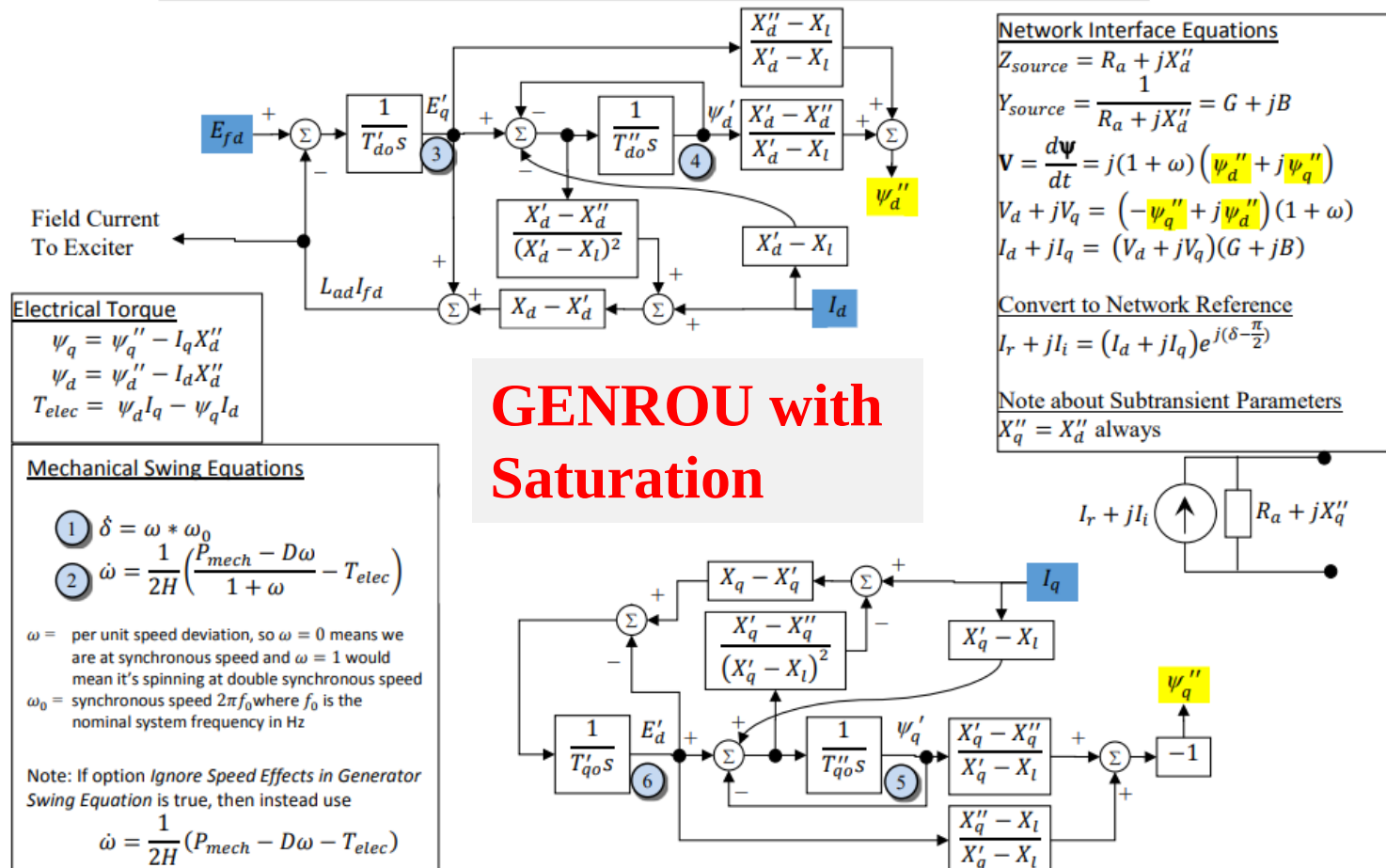
- When implementing saturation models in code, it is important to recognize that the function is meant to be positive, so negative values are not allowed
- In large cases one is almost guaranteed to have special cases, sometimes caused by user typos
 - What to do if $Se(1.2) < Se(1.0)$?
 - What to do if $Se(1.0) = 0$ and $Se(1.2) \neq 0$
 - What to do if $Se(1.0) = Se(1.2) \neq 0$

Where in Equations do you put Saturation: GENROU Example



Machine Model GENROU

Machine Model GENROU
Solid Rotor Generator represented by equal mutual inductance rotor modeling

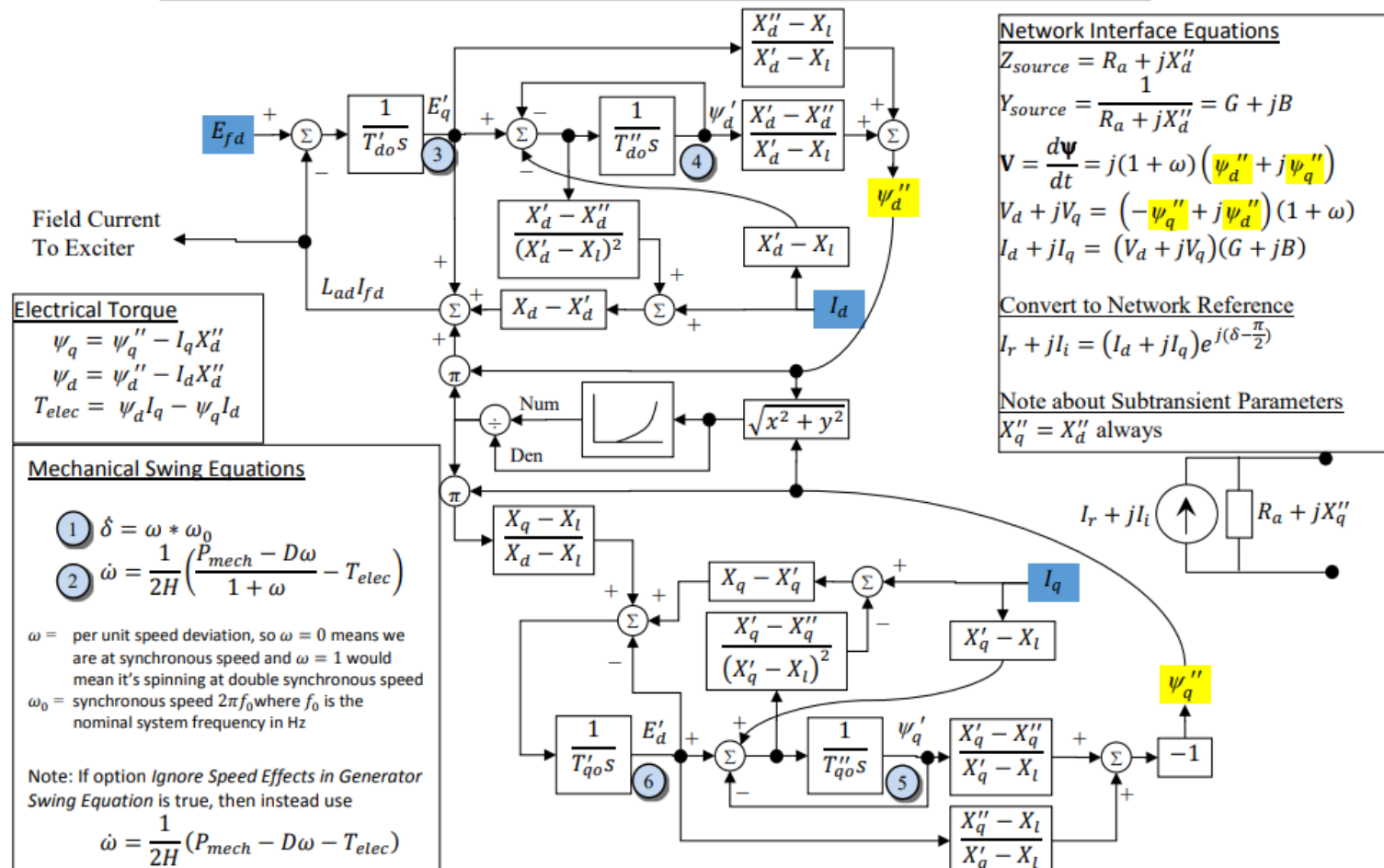


Where in Equations do you put Saturation: GENROU Example



Machine Model GENROU

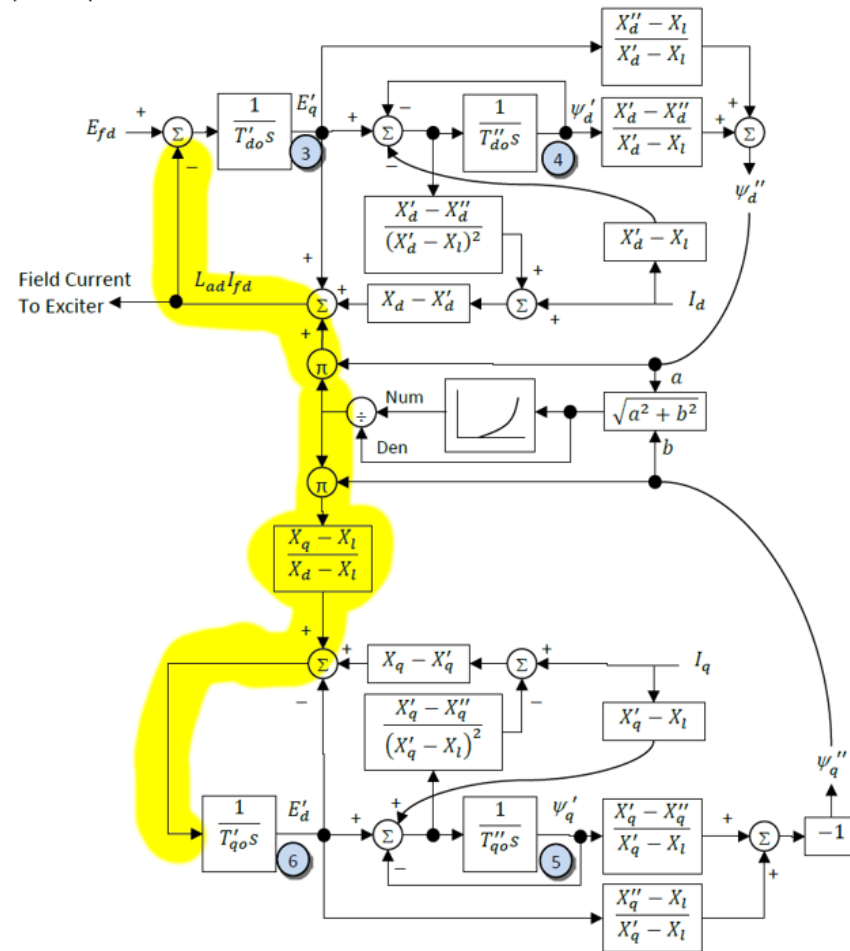
Machine Model GENROU
Solid Rotor Generator represented by equal mutual inductance rotor modeling



Saturation is being Added to the Sub-transient derivative



- Saturation is Added to the Field Current
 - This makes sense especially because of the test that is used to find the saturation function
- The Open Circuit Saturation Test only finds saturation on D-Axis
 - We know there is also saturation on Q-Axis
 - We simply assume it is the proportional to the mutual inductance



My Personal Assessment of GENROU saturation



- Why was it done this way?
- This exactly matches the test which is done
- A steady state open circuit test is done by varying the field current and seeing what the terminal voltage of the synchronous machine does
 - This means $I_d = I_q = 0$ (zero stator current)
 - In this open circuit test the q-axis fluxes are all zero! (ψ_q) so we are only measuring the d-axis saturation
 - Varying the terminal voltage will vary the d-axis flux (ψ_d'')
 - If you have a curve showing you the extra field current needed, in 1970 it made perfect sense to just add in that “extra” field current right into the block diagram
 - This is exactly what GENROU represents

Scaling of Saturation on q-Axis



- Mutual Inductance is something we skipped over earlier in our per unit parameterization choices (Page 30 – 42 of Sauer/Pai book)
- $X_d = X_{md} + X_l \rightarrow X_{md} = X_d + X_l$
- $X_q = X_{mq} + X_l \rightarrow X_{mq} = X_q + X_l$
 - X_d and X_q = synchronous reactance on d-axis and q-axis
 - X_{md} and X_{mq} : mutual inductance on the d-axis and q-axis (this is the part inside the iron core so it saturates)
 - X_l : this is the leakage inductance. This is the inductance from the air gap, so we assume air does not saturate
- The GENROU choice of scaling simply says the same relative amount of saturation occurs on both d and q axis
- $\frac{X_{mq}}{X_{md}} = \frac{X_q + X_l}{X_d + X_l}$: Scaling term to convert d-axis to q-axis

Implications of modeling saturation with addition



- There were some fundamental relationships between the 4 rotor flux variables and how they relate to one another
 - By adding to ONLY two of those flux terms we have disrupted those relationships
- Saturation does NOT impact the network boundary equations at all
 - As long as we require that $X_d'' = X_q''$, a simple circuit equation can be used at network boundary
 - $X_d'' \neq X_q''$ is called transient saliency and is not allowed in GENROU and GENSAL models
 - This makes it much easier on software writers and is likely a big reason why in 1970 this would have been picked as approximation
- Saturation is only a function of the flux and thus the terminal voltage of the synchronous machine
 - Saturation is NOT a function of armature current

An Aside

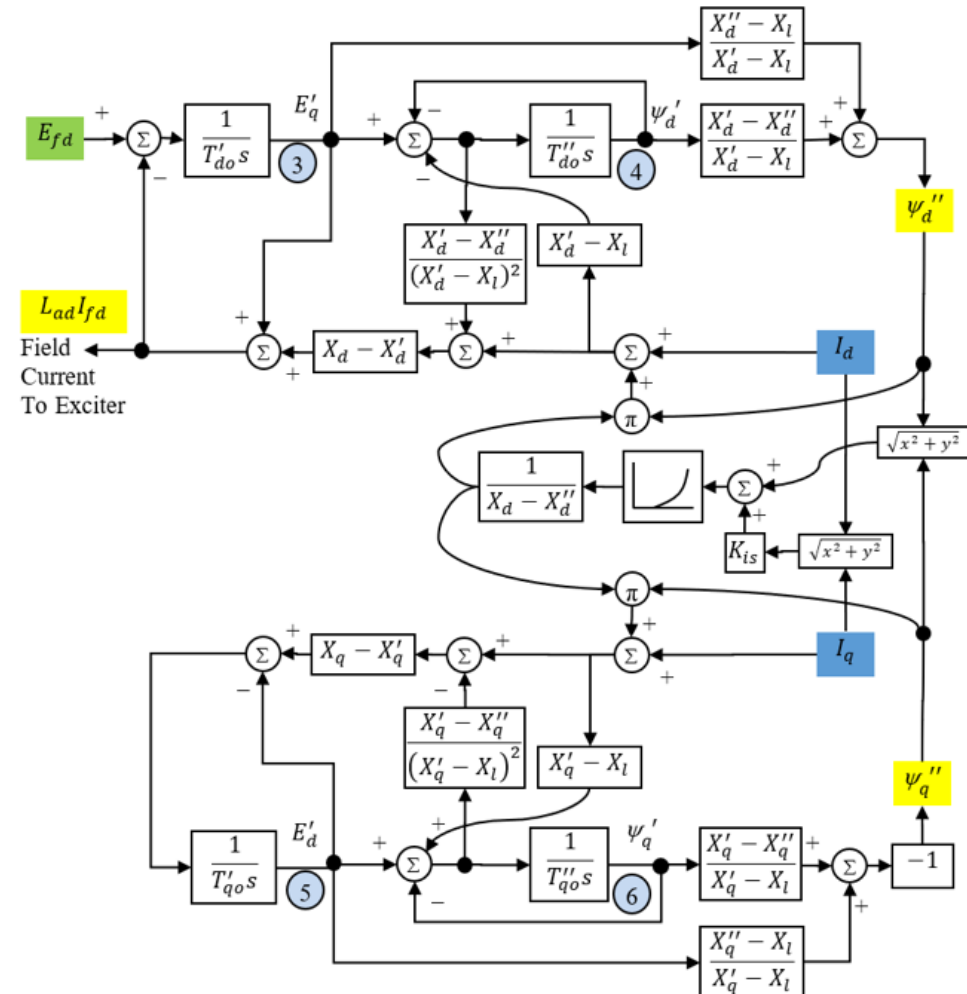


- Beyond the scope of this presentation, but there are some fundamental theoretical flaws with the treatment of GENROU as well
 - Peter W. Sauer “Constraints on Saturation Modeling in AC Machines”, IEEE Transactions on Energy Conversion, Vo. 7. No. 1, March 1992, pp. 161 – 167
 - This paper discusses the fundamental assumption that the “coupling field” between the mechanical and electrical systems is a conservative (lossless) field
 - That assumption puts some theoretical constraints on how the saturation should be applied to the equations
 - A ton of calculus and algebra and you would get what I call the “GENPWS” (for Peter W. Sauer)

An Aside

“GENPWS” for Peter. W. Sauer

- Still uses addition
 - But the addition is made to the input stator current instead
 - Increasing stator current being fed into dynamic model increases the field voltage
 - It's a lot of math, but this makes some sense
 - Does not change the fundamental relationships between all the rotor quantities

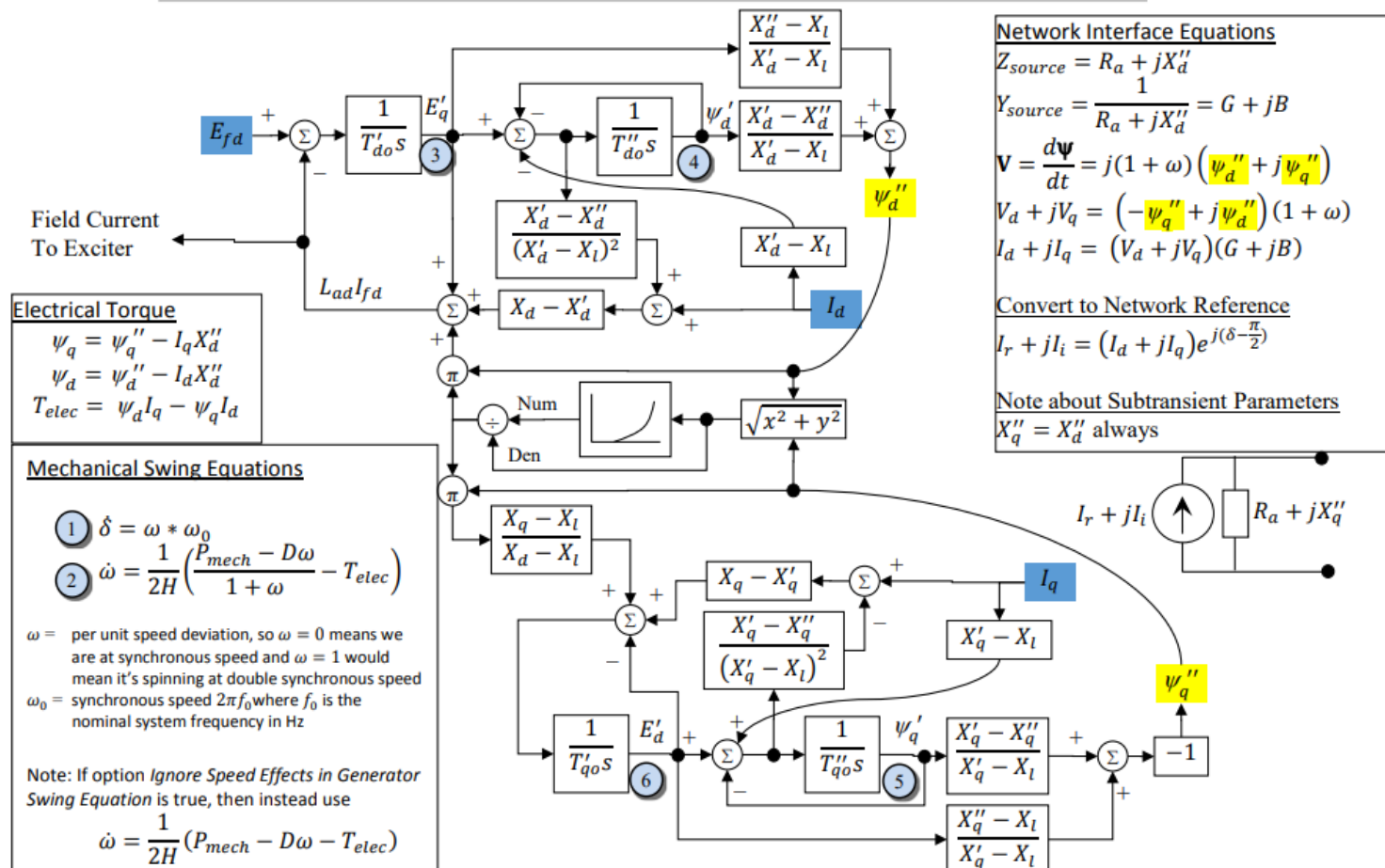


Initialization of GENROU Model



Machine Model GENROU

Machine Model GENROU
Solid Rotor Generator represented by equal mutual inductance rotor modeling



Initialization of GENROU with Saturation



- Write 5 equations as follows from Block Diagram at Steady State (inputs to integrators = 0)

1. $(X'_d - X_l)I_d + \psi'_d - E'_q = 0$

Sum inputs integrator for State 4

2. $\psi''_d - \left(\frac{X''_d - X_l}{X'_d - X_l}\right)E'_q - \left(\frac{X'_d - X''_d}{X'_d - X_l}\right)\psi'_d = 0$

Final summation block resulting in ψ''_d

3. $(X_q - X'_q)I_q + \psi''_q \left(\frac{X_q - X_l}{X'_d - X_l}\right) Sat(\psi'') - E'_d = 0$

Sum inputs integrator for State 6

4. $(X'_q - X_l)I_q - \psi'_q + E'_d = 0$

Sum inputs integrator for State 5

5. $-\psi''_q - \left(\frac{X''_q - X_l}{X'_q - X_l}\right)E'_d - \left(\frac{X'_q - X''_q}{X'_q - X_l}\right)\psi'_q = 0$

Final summation block resulting in ψ''_q

- Know $Sat(\psi'')$ already (next slide)
- Rotor Angle (δ) is in there too because we need it to perform the translation from network to machine reference frame

Getting Saturation Function at Initialization



- We can get the internal voltage on the network reference frame directly
 - $V_r + V_i = (V_{rterm} + jV_{iterm}) + (R_a + jX_d'')(I_r + jI_i)$
 - $V_r = V_{rterm} + R_a I_r - X_d'' I_i$
 - $V_i = V_{iterm} + R_a I_i + X_d'' I_r$
- From this we get the saturation function because the reference frame conversion only rotates, therefore the magnitude does not change!

$$Sat(\psi'') = Sat(|\psi_d'' + j\psi_q''|) = Sat(|V_q - jV_d|) = Sat\left(\left|(V_d + jV_q)e^{j(\delta - \frac{\pi}{2})}\right|\right) = Sat(|(V_r + jV_i)|)$$

- $Sat(\psi'') = Sat(|(V_r + jV_i)|)$
- From here it's a bit of clever algebra, but you can get the initial rotor angle

Derivation of initial Rotor Angle for GENROU with Saturation



Use equation 4 to solve for	$\psi'_q = (X'_q - X_l)I_q + E'_d$
Substitute this into equation 5	$-\psi''_q - \left(\frac{X''_q - X_l}{X'_q - X_l}\right) E'_d - \left(\frac{X'_q - X''_q}{X'_q - X_l}\right) [(X'_q - X_l)I_q + E'_d] = 0$
Expand this to	$-\psi''_q - \left(\frac{X''_q - X_l}{X'_q - X_l}\right) E'_d - \left(\frac{X'_q - X''_q}{X'_q - X_l}\right) E'_d - \left(\frac{X'_q - X''_q}{X'_q - X_l}\right) (X'_q - X_l)I_q = 0$
Simplify this to	$-\psi''_q - E'_d - (X'_q - X''_q)I_q = 0$
Flip signs to get	$+\psi''_q + E'_d + (X'_q - X''_q)I_q = 0$
Use equation 3 to solve for	$E'_d = +\psi''_q \left(\frac{X_q - X_l}{X_d - X_l}\right) Sat(\psi'') + (X_q - X'_q)I_q$
Substitute this into previous equation to get	$+\psi''_q + \psi''_q \left(\frac{X_q - X_l}{X_d - X_l}\right) Sat(\psi'') + (X_q - X'_q)I_q + (X'_q - X''_q)I_q = 0$
This then simplifies to	$+\psi''_q \left[1 + \left(\frac{X_q - X_l}{X_d - X_l}\right) Sat(\psi'')\right] + (X_q - X''_q)I_q = 0$
Now we have values on the d/q axis, so let us convert them to the network reference frame using the equations and also define the term K_{sat} as follows.	$\psi''_q = -V_d = -V_r \sin(\delta) + V_i \cos(\delta)$ $I_q = I_r \cos(\delta) + I_i \sin(\delta)$ $K_{sat} = \left[1 + \left(\frac{X_q - X_l}{X_d - X_l}\right) Sat(\psi'')\right]$

Derivation of Initial Rotor Angle

Continued



This gives us	$[-V_r \sin(\delta) + V_i \cos(\delta)]K_{sat} + (X_q - X_q'')[I_r \cos(\delta) + I_i \sin(\delta)] = 0$
Group the sine and cosine terms to get	$[-K_{sat}V_r + I_i(X_q - X_q'')] \sin(\delta) + [K_{sat}V_i + I_r(X_q - X_q'')] \cos(\delta) = 0$
Turn this into the tangent function	$\frac{\sin(\delta)}{\cos(\delta)} = \tan(\delta) = \frac{[K_{sat}V_i + I_r(X_q - X_q'')]}{[K_{sat}V_r - I_i(X_q - X_q'')]}$
Then remember how the V_r and V_i terms were defined and substitute those back in	$\tan(\delta) = \frac{[K_{sat}(V_{iterm} + R_a I_i + X_d'' I_r) + I_r(X_q - X_q'')]}{[K_{sat}(V_{rterm} + R_a I_r - X_d'' I_i) - I_i(X_q - X_q'')]}$
Arrange terms here gain and you get	$\tan(\delta) = \frac{[K_{sat}V_{iterm} + K_{sat}R_a I_i + (K_{sat}X_d'' + X_q - X_q'')I_r]}{[K_{sat}V_{rterm} + K_{sat}R_a I_r - (K_{sat}X_d'' + X_q - X_q'')I_i]}$

GENROU without Saturation is Easier



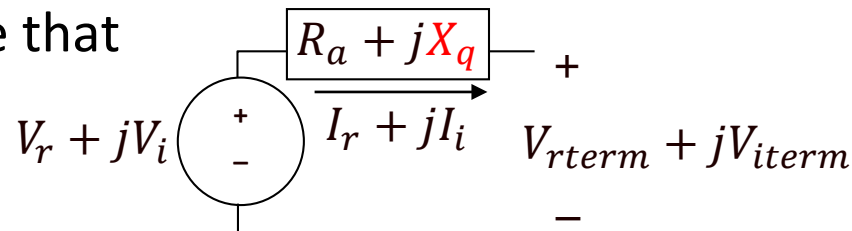
- Rotor Angle can be obtained from network reference frame quantities

$$\delta = \tan^{-1} \left(\frac{K_{sat} V_{iterm} + K_{sat} R_a I_i + (K_{sat} X_d'' + X_q - X_q'') I_r}{K_{sat} V_{rterm} + K_{sat} R_a I_r - (K_{sat} X_d'' + X_q - X_q'') I_i} \right)$$

- If $K_{sat} = 1.0$, that means there is no saturation
 - The rotor angle then simplifies to

$$\delta = \tan^{-1} \left(\frac{V_{iterm} + R_a I_i + X_q I_r}{V_{rterm} + R_a I_r - X_q I_i} \right)$$

- Sauer/Pai book page 51 will show finding the angle of the internal voltage using the following circuit
- Note: It's kind of weird because that is NOT the network interface as it uses X_q instead of X_q''



Initialization of the Remainder



- Rotor Angle known, continue with algebra

- $V_d = V_r \sin(\delta) - V_i \cos(\delta)$

convert to dq axis using guess for δ

- $V_q = V_r \cos(\delta) + V_i \sin(\delta)$

convert to dq axis using guess for δ

- $I_d = I_r \sin(\delta) - I_i \cos(\delta)$

convert to dq axis using guess for δ

- $I_q = I_r \cos(\delta) + I_i \sin(\delta)$

convert to dq axis using guess for δ

- $\psi_q'' = -\frac{V}{d} 1_{+\omega}$

really we can ignore ω as it's zero

- $\psi_d'' = +\frac{V}{q} 1_{+\omega}$

really we can ignore ω as it's zero

- $\psi_d' = \psi_d'' - I_d(X_d'' - X_l)$

State 4. Algebra from summation blocks for ψ_d''

- $E_q' = \psi_d' + I_d(X_d' - X_l)$

State 3. Enforce zero input to state 4 integral block

- $E_d' = I_q(X_q - X_q')$

State 6. Enforce zero input to state 6 integral block

- $\psi_q' = E_d' + I_q(X_q' - X_l)$

State 5. Enforce zero input to state 5 integral block

Machine Model Reactance Validation



- For synchronous machine models, there are d-axis and q-axis reactance values
 - Synchronous reactance : X_d and X_q
 - Transient reactance : X'_d and X'_q
 - Sub-transient reactance : X''_d and X''_q
 - Leakage reactance : X_l
- The following two relationships must be satisfied (physically impossible to violate)
 - $X_l \leq X''_q \leq X'_q \leq X_q$
 - $X_l \leq X''_d \leq X'_d \leq X_d$
 - These types of model errors are not uncommon

Machine Reactance

Auto-Correction in PowerWorld



- When machine reactance model errors are found and auto-correction is applied, the following changes will be applied to the data
 - If $X_{q'} > X_q$ then $X_{q'} = 0.8X_q$
 - If $X_{d'} > X_d$ then $X_{d'} = 0.8X_d$
 - If $X_{q''} > X_{q'}$ then $X_{q''} = 0.8X_{q'}$
 - If $X_{d''} > X_{d'}$ then $X_{d''} = 0.8X_{d'}$
 - If $X_l > X_{q''}$ then $X_l = 0.8X_{q''}$
 - If $X_l > X_{d''}$ then $X_l = 0.8X_{d''}$

GENTPF/GENTPJ



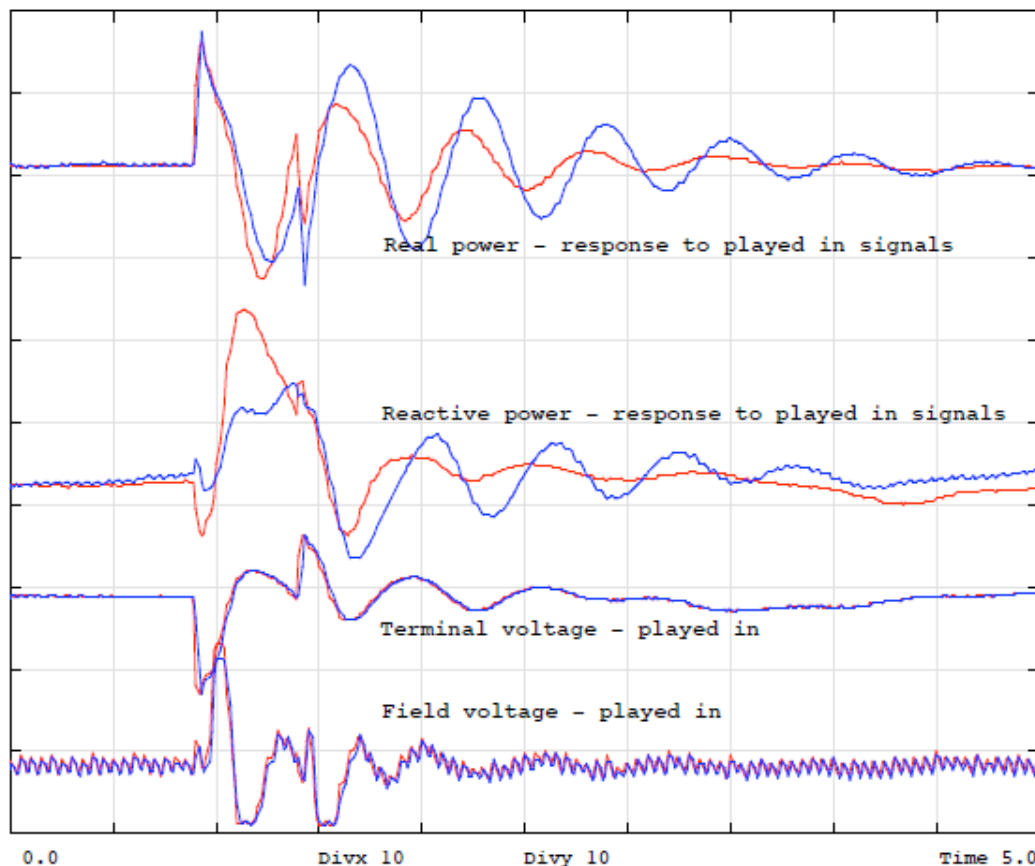
- GENTPF Modifies the treatment of Saturation
 - Instead of being introduced as addition, consider treating the input parameters as the “unsaturated parameters”
 - (X_d , X_q , X_{dp} , X_{qp} , X_{dpp} , X_{qpp} , T_{dop} , T_{dopp} , T_{qop} , T_{qopp})
 - Then apply multiplication to account for saturation
 - This has advantage of applying saturation to all the equations simultaneously
 - GENTPJ also introduces the impact of the stator current on saturation.
 - Empirical testings show that increases in stator current increases saturation.
 - Again, we skipped all this, but the various parameters we derived
- GENTPF/GENTPJ however, does make a small change to the dynamic equations that may not have been quite right. We’re working on testing new models

GENTPF and GENTPJ Models



- These models were introduced in 2009 to provide a better match between simulated and actual system results for salient pole machines
 - Desire was to duplicate functionality from old BPA TS code
 - Allows for subtransient saliency ($X_q'' \neq X_d''$)
 - Can also be used with round rotor, replacing GENSAL and GENROU
- Useful reference is available at below link; includes all the equations, and saturation details

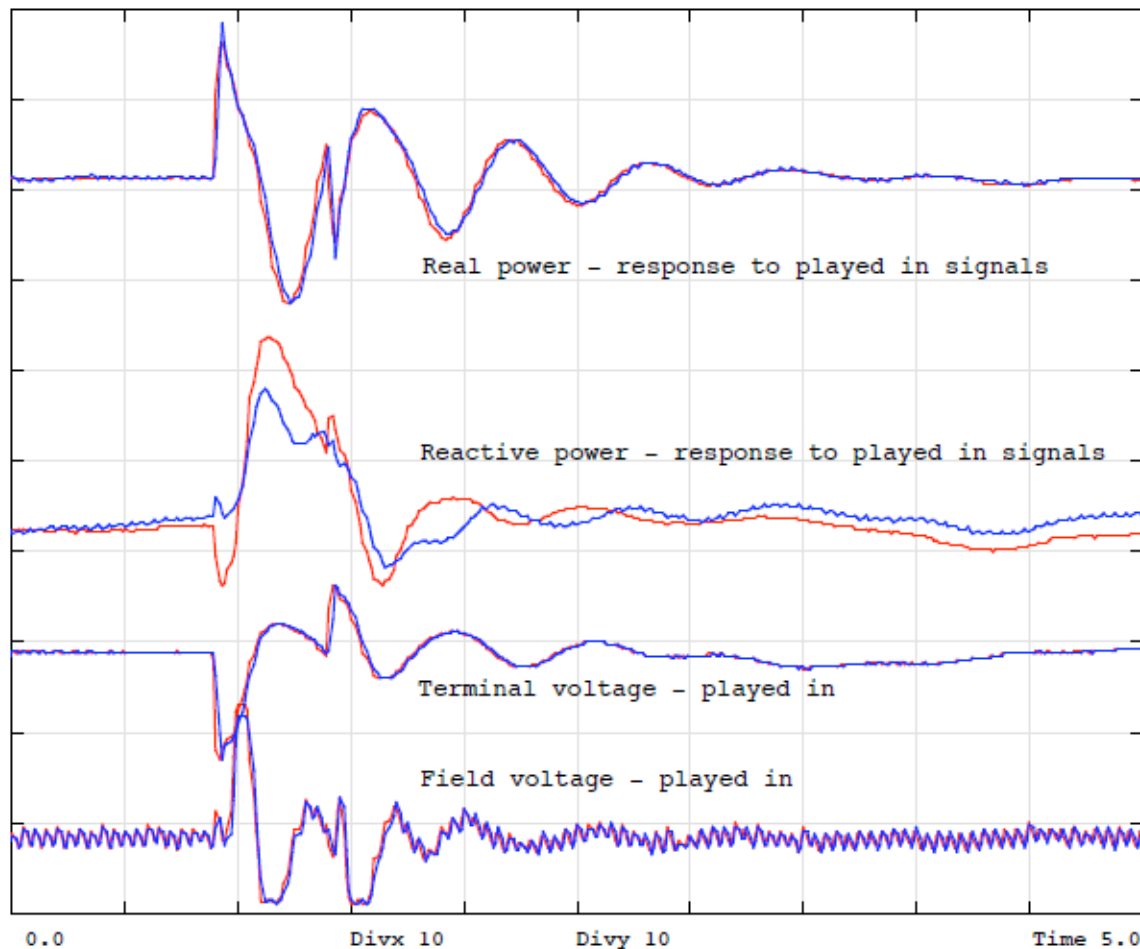
Motivation for the Change: GENSAL Actual Results



**Chief Joseph
disturbance playback
GENSAL
BLUE = MODEL
RED = ACTUAL
(Chief Joseph is a
2620 MW hydro
plant on the
Columbia River in
Washington)**

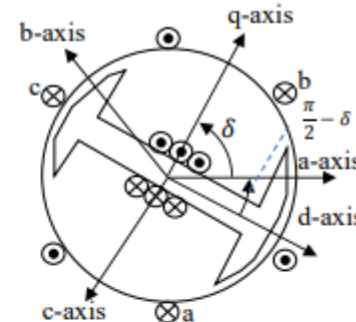
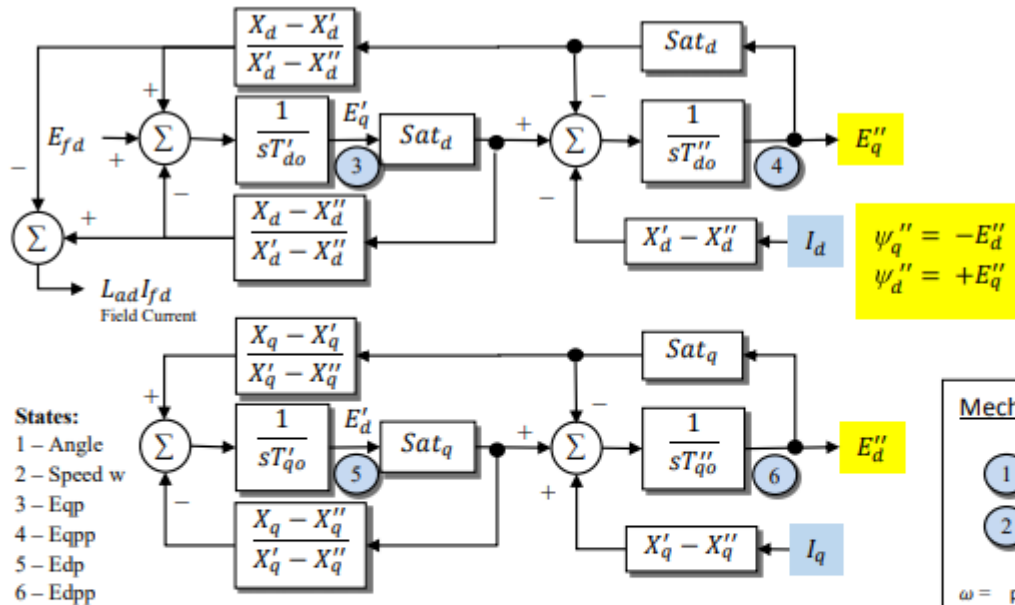
Image source :<https://www.wecc.biz/library/WECC%20Documents/Documents%20for%20Generators/Generator%20Testing%20Program/gentpj%20and%20gensal%20morel.pdf>

GENTPJ Results



**Chief Joseph
disturbance
playback
GENTPJ
BLUE = MODEL
RED = ACTUAL**

GENTPF and GENTPJ Models



$$\psi_q'' = -E_d''$$

$$\psi_d'' = +E_q''$$

$$\psi_{ag} = \sqrt{(V_{qterm} + I_q R_a + I_d X_l)^2 + (V_{dterm} + I_d R_a - I_q X_l)^2}$$

$$\text{D-Axis: } Sat_d = 1 + \text{SaturationFunction} \left(\psi_{ag} + K_{is} \sqrt{I_d^2 + I_q^2} \right)$$

$$\text{Q-Axis: } Sat_q = 1 + \frac{X_q}{X_d} \text{SaturationFunction} \left(\psi_{ag} + K_{is} \sqrt{I_d^2 + I_q^2} \right)$$

Mechanical Swing Equations

$$1 \quad \dot{\delta} = \omega * \omega_0$$

$$2 \quad \dot{\omega} = \frac{1}{2H} \left(\frac{P_{mech} - D\omega}{1 + \omega} - T_{elec} \right)$$

ω = per unit speed deviation, so $\omega = 0$ means we are at synchronous speed and $\omega = 1$ would mean it's spinning at double synchronous speed
 ω_0 = synchronous speed $2\pi f_0$ where f_0 is the nominal system frequency in Hz

Note: If option *Ignore Speed Effects in Generator Swing Equation* is true, then instead use

$$\dot{\omega} = \frac{1}{2H} (P_{mech} - D\omega - T_{elec})$$

Most of WECC machine models are now GENTPF or GENTPJ

If nonzero, K_{is} typically ranges from 0.02 to 0.12

Network Equation for GENTPF/GENTPJ



- Network Equations can NOT use circuit equation
- Results in some complexity we don't discuss here

Saturated Subtransient Reactances

$$X''_{dsat} = \frac{X''_d - X_l}{Sat_d} + X_l$$

$$X''_{qsat} = \frac{X''_q - X_l}{Sat_q} + X_l$$

Electrical Torque

$$\psi_q = \psi_q'' - I_q X''_{qsat} = -E_d'' - I_q X''_{qsat}$$

$$\psi_d = \psi_d'' - I_d X''_{dsat} = +E_q'' - I_d X''_{dsat}$$

$$T_{elec} = \psi_d I_q - \psi_q I_d$$

Network Interface Equations

$$V_d + jV_q = (E_d'' + jE_q'')(1 + \omega)$$

Because $X''_{qsat} \neq X''_{dsat}$ we cannot use a circuit model for network interface equations and must instead directly use the following.

$$V_{dterm} = V_d - R_a I_d + X''_{qsat} I_q$$

$$V_{qterm} = V_q - X''_{dsat} I_d - R_a I_q$$

Theoretical Justification for GENTPF and GENTPJ



- In the GENROU and GENSAL models saturation shows up purely as an additive term of E_q' and E_d'
 - Saturation does not come into play in the network interface equations and thus with the assumption of $X''_{qsat} = X''_{dsat}$ a simple circuit model can be used
- The advantage of the GENTPF/J models is saturation really affects the entire model, and in this model it is applied to all the inductance terms simultaneously
 - This complicates the network boundary equations, but since these models are designed for $X''_{qsat} \neq X''_{dsat}$ there is no increase in complexity

GENTPF/GENTPJ: However?



- If you look at the differential equations, they are different!
 - In particular notice that the leakage reactance is NOT on the block diagram for the differential equations
 - The GENTPF/GENTPJ differential equations represent what would happen to the GENROU equations if you make the approximation on each axis
 - $X_l = X_d''$ on the d-axis differential equations
 - $X_l = X_q''$ on the q-axis differential equations
- This has some undesired effects on the transient response
 - Quincy Wang at PowerTech (now at B.C. Hydro) first pointed this out to me a few years ago

Why does this even matter?



- GENROU and GENSAL models date from 1970, and their purpose was to replicate the dynamic response the synchronous machine
 - They have done a great job doing that
- Weaknesses of the GENROU and GENSAL model has been found to be with matching the field current and field voltage measurements
 - Field Voltage/Current may have been off a little bit, but that didn't effect dynamic response
 - It just shifted the values and gave them an offset
- Shifted/Offset field voltage/current didn't matter too much in the past

Over and Under Excitation Limiters



- Traditionally our industry has not modeled over excitation limiters (OEL) and under excitation limiters (UEL) in transient stability simulation
 - The Mvar outputs of synchronous machines during transients likely do go outside these bounds in our existing simulations
 - Our Simulation haven't been modeling limits being hit anyway, so the overall dynamic response isn't impacted
- If the industry wants to start modeling OEL and UEL, then we need to better match the field voltage and currents
 - Otherwise we're going to be hitting these limits when in real life we are not

GENQEC



- Saurav Mohapatra and Jamie Weber at PowerWorld Corporation have been working with Quincy Wang at BC Hydro on a “GENQEC”
- We agreed on two main points!
 - Saturation function should be applied to all input parameters by multiplication
 - This also ensures a conservative coupling field assumption of Peter W. Sauer paper from 1992
 - Same multiplication should be applied to both d-axis and q-axis terms (assume same amount of saturation on both)
- Results in differential equations that are nearly the same as GENROU
 - Scales the inputs and outputs, and effects time constants
- Network Interface Equation is same as GENTPF/J

Applying Saturation to the Dynamic Equations



- From machine design and analysis, these are the reactance values that saturate: X_{md} , X_{fd} , X_{1d} , and X_{mq} , X_{1q} , X_{2q}
 - Go back to Sauer/Pai book to see what these are
- Assume leakage reactance does NOT saturate: X_l
- In transient stability, we use transformed constants:
 X_d , X'_d , X''_d , and X_q , X'_q , X''_q
- GENTPF/GENTPJ used following in network boundary equations

$$X''_{dsat} = \frac{X''_d - X_l}{\text{Sat}_d} + X_l$$

$$X''_{qsat} = \frac{X''_q - X_l}{\text{Sat}_q} + X_l$$

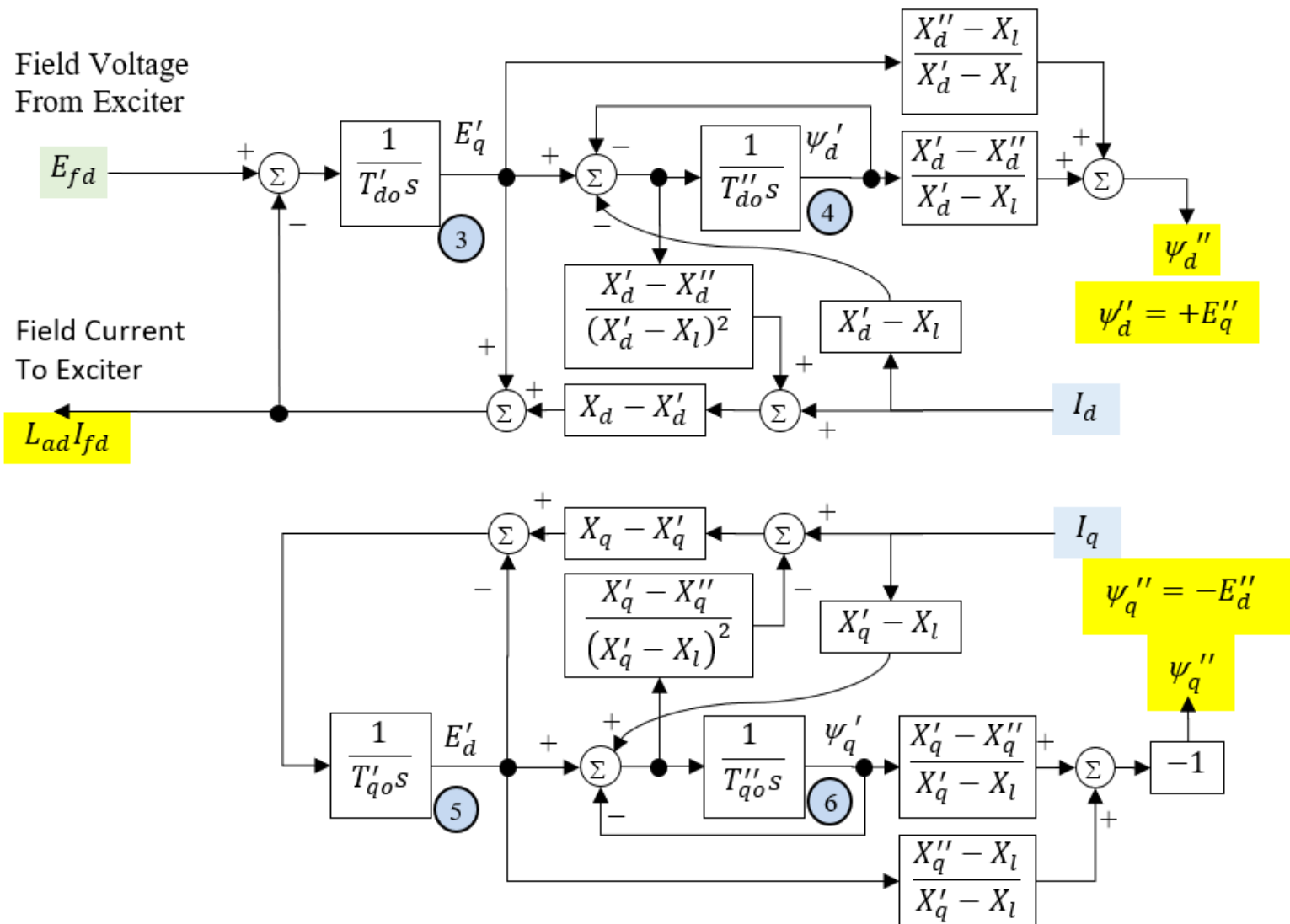
GENQEC – Extend concept used in GENTPJ algebraic network equations



	d-axis	q-axis
Reactance Values	$X''_{dsat} = \frac{X''_d - X_l}{Sat_d} + X_l$ $X'_{dsat} = \frac{X'_d - X_l}{Sat_d} + X_l$ $X_{dsat} = \frac{X_d - X_l}{Sat_d} + X_l$	$X''_{qsat} = \frac{X''_q - X_l}{Sat_q} + X_l$ $X'_{qsat} = \frac{X'_q - X_l}{Sat_q} + X_l$ $X_{qsat} = \frac{X_q - X_l}{Sat_q} + X_l$
Time Constants (are a function of reactance values)	$T'_{dosat} = \frac{T'_{do}}{Sat_d}$ $T''_{dosat} = \frac{T''_{do}}{Sat_d}$	$T'_{qosat} = \frac{T'_{qo}}{Sat_q}$ $T''_{qosat} = \frac{T''_{qo}}{Sat_q}$
Exciter Interface Signals	$E_{fdsat} = \frac{E_{fd}}{Sat_d}$ $X_{mdsat} I_{fd} = (X_{dsat} - X_l) I_{fd} = \frac{X_d - X_l}{Sat_d} I_{fd}$	

GENROU Block Diagram

Without Saturation

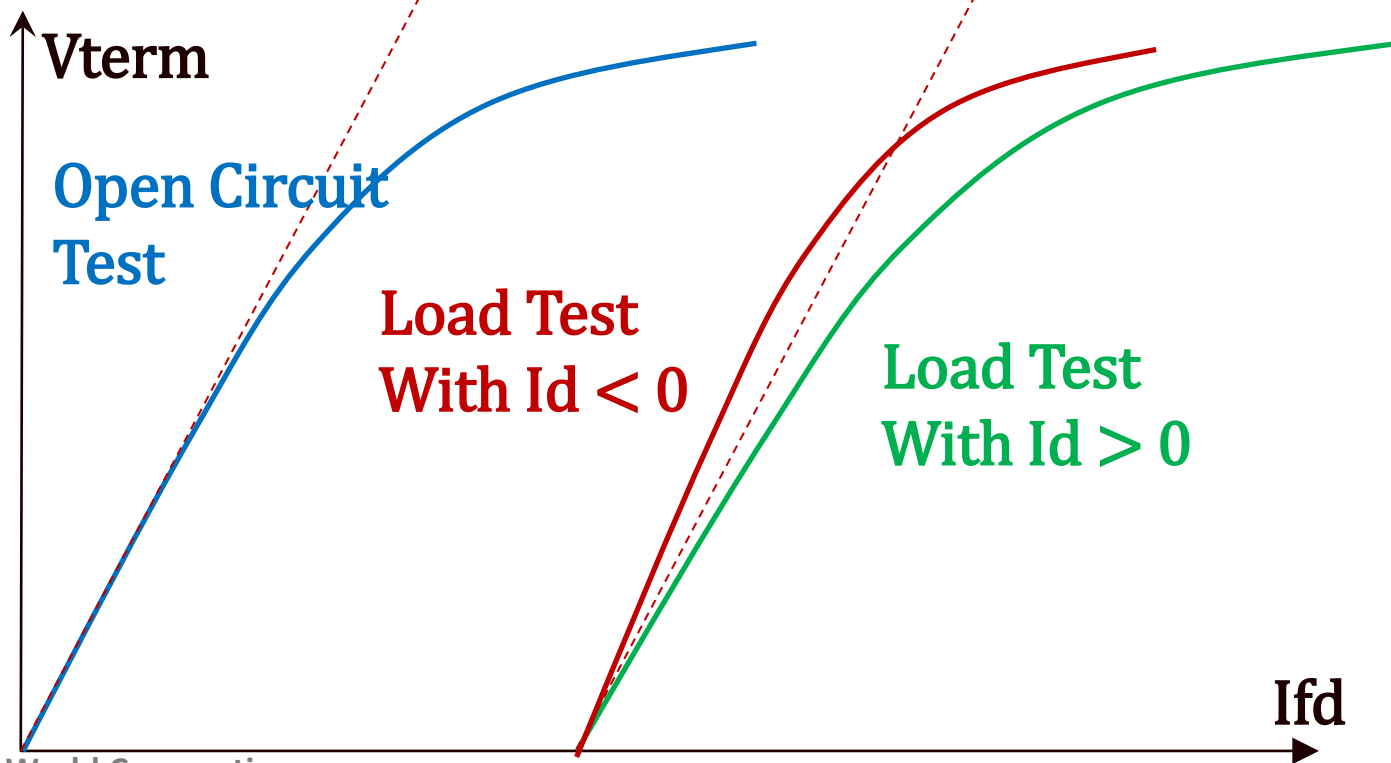




GENQEC



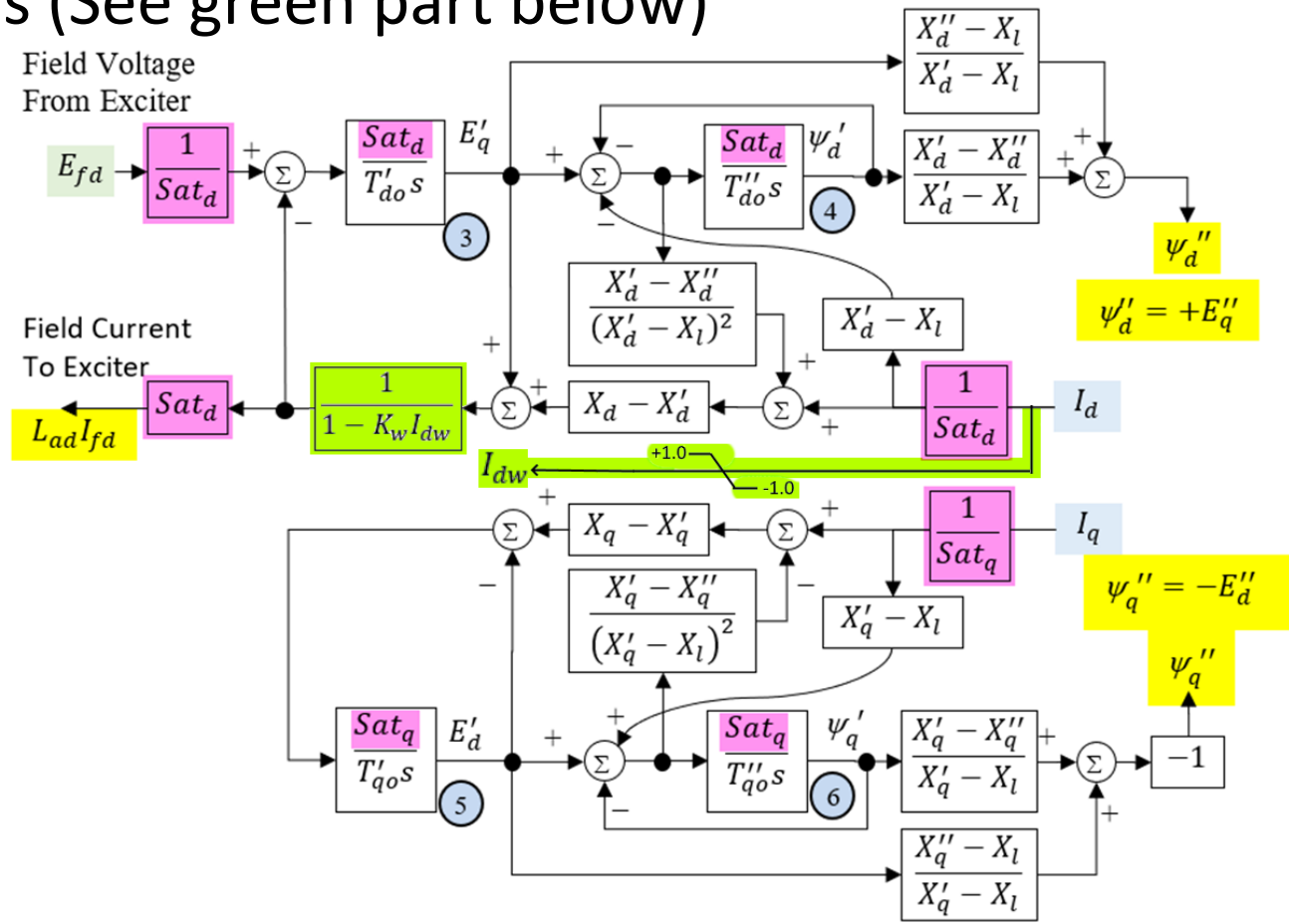
- Quincy tests and other tests showed that the slope of the V_{term} vs. I_{fd} curve changed as the generator is loaded.
 - This slope changes occurred immediately
 - This is what we are calling “compensation”



GENQEC



- Use GENTPF network equation plus add K_w term that increases field voltage as the stator current increases (See green part below)



Comment about all these Synchronous Machine Models



- We are improving these models
 - Does not mean the old models were useless
- All these models have the same input parameter names, but that does not mean they are exactly the same
 - Input parameters are tuned for a particular model
 - It is NOT appropriate to take all the parameters for GENROU and just copy them over to a GENTPJ model and call that your new model
 - When performing a new generator testing study, that is the time to update the parameters

Status of these models



- Applying saturation using multiplication is definitely better theoretically
 - Maintains the relationships between the rotor fluxes as they all scale the same
- Multiplication saturation has shown better experimental results
 - GENTPF/GENTPJ results are related to this
 - Quincy at BC Hydro has confirmed good fits with GENQEC
 - Saurav at PowerWorld confirmed good fits to test data using an earlier version of this model we called GENTPW
- The GENQEC model's Kw term is another experimentally determined parameter to add to the model
 - It removes the need for the Kis parameter, as the addition of the compensation factor gives a similar effect of stator current impacting the field current and voltage

PowerWorld's hope for GENQEC



- PowerWorld had hopes of figuring out some theoretical relationship from cross saturation (currents on the d-axis creating saturation on the q-axis and vice-versa) that would explain the need for Kw term and lead to both good experimental results and better theoretical justification
- Status: We give up!
- GENQEC Kw term is explainable from the experimental test data.
- We leave it to another Ph.D. student someday to help explain better why this is needed

Input Parameters in Dialog

- Stability Tab
- Machine Model Tab

The screenshot displays the 'Generator Information for Present' dialog box. The 'Stability' tab is selected, and the 'Machine Models' sub-tab is active. The dialog shows various input fields for generator parameters, including bus information, status, energized state, fuel type, unit type, and a detailed list of machine model parameters.

Generator Information for Present

Bus Number: 14931
Bus Name: [blank]
ID: [blank]
Area Name: [blank]
Labels ...: [blank]
Generator MVA Base: 1559.00

Status:
☐ Open
☒ Closed
Energized:
☐ NO (Offline)
☒ YES (Online)
Fuel Type: Unknown
Unit Type: CA (Combined Cycle Steam Part)

Power and Voltage Control | Costs | OPF | Faults | Owners, Area, etc. | Custom | **Stability**

Machine Models | Exciters | Governors | Stabilizers | Other Models | Step-up Transformer | Terminal and State

Insert | Delete | Gen MVA Base: 1559.0 | Show Block Diagram | Create VCurve

Type: Active - GENROU ☒ Active (only one may be active) | Set to Defaults

Parameters
PU values shown/entered using device base of 1559.0 MVA

H	3.83100	Xl	0.22000	XComp	0.07000
D	0.00000	Tdop	10.00000		
Ra	0.00360	Tqop	0.48000		
Xd	2.19000	Tdopp	0.03300		
Xq	1.86000	Tqopp	0.05500		
Xdp	0.37000	S1	0.18100		
Xqp	0.56400	S12	0.93900		
Xdpp	0.28000	RComp	0.00000		

OK | Save | Save to Aux | Cancel | Help | Print

Seeing Terminal Bus values and Exciter/Governor Setpoint



- On Stability Tab, then Terminal and State, then Bus/Setpoint Values

Network Reference Frame

The screenshot displays the PowerWorld software interface. At the top, the 'Generator MVA Base' is set to 1559.00 and the 'Unit Type' is 'CA (Combined Cycle Steam Part)'. The 'Stability' tab is selected, and within it, the 'Terminal and State' sub-tab is active. The 'Bus/Setpoint Values' sub-tab is also selected, showing the following data:

Terminal Voltage and Current into Bus (Polar)		Power Values into Bus		
	Magnitude (pu)	Angle (deg)		
Voltage	0.9967	18.1423	MW	1372.197
Current	14.1430	4.898	Mvar	322.965
			MVA	1409.691

Terminal Voltage and Current into Bus (Polar)		Model Setpoint Values		
	Real (pu)	Imaginary (pu)		
Voltage	0.9472	0.3104	Exciter Setpoint (pu)	0.9862
Current	14.0914	1.2076	Governor Setpoint (pu)	0.8831

Step-Up Transformer Loss	
MW Loss	
0.000	

Seeing Terminal Values



- On Stability Tab, then Terminal and State, then Terminal Values

Generator MVA Base 1559.00 Unit Type CA (Combined Cycle Steam Part) ▾

Power and Voltage Control Costs OPF Faults Owners, Area, etc. Custom **Stability**

Machine Models Exciters Governors Stabilizers Other Models Step-up Transformer **Terminal and State**

Simulation Time (seconds) Initialized at 0.000000 Save Two Bus Equivalent

PU values shown/entered using device base of 1559.0 MVA ▾

Bus/Setpoint Values **Terminal Values** Machine States Exciter States Governor States Stabilizer States

Terminal Voltage and Current		Internal Angle		Field	
	Magnitude (pu)	Angle (deg)			
Voltage	0.9967	18.1423	Internal Angle Delta (deg)	62.4971	Voltage (pu)
Current	0.9072	4.898	Freq. Deviation omega (rad/sec)	0.0000	2.7970
			Acceleration (rad/sec^2)	0.0000	Current (pu)
					2.7970

Power Values		Direct and Quadrature Axis Values	
		Direct Axis	Quadrature Axis
Mechanical Input MW	1376.819	Terminal Voltage (pu)	0.6968 0.7127
Accelerating MW	0.003	Terminal Current (pu)	0.7660 0.4861
Terminal MW	1372.197	Internal Voltage (pu)	0.5635 0.9289
Terminal Mvar	322.965		
Terminal MVA	1409.691		

Machine Reference Frame

δ

E_{fd}

$L_{ad} I_{fd}$

V_{dterm} V_{qterm}

I_d I_q

V_d

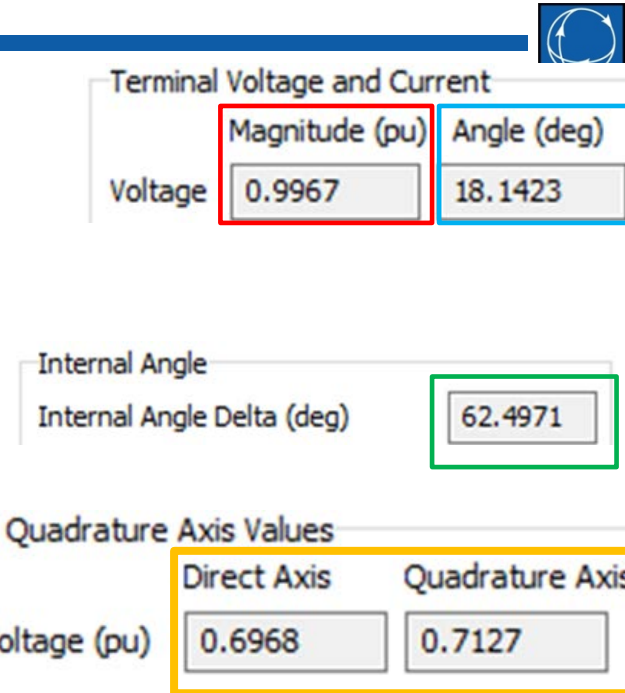
V_q

Network Reference Frame

- Let's do example
 - $V_r = 0.9967 \cos(18.1423^\circ) = 0.947150$
 - $V_i = 0.9967 \sin(18.1423^\circ) = 0.310351$
 - $\sin(\delta) = \sin(62.4971^\circ) = 0.886987$
 - $\cos(\delta) = \cos(62.4971^\circ) = 0.461794$
- Transform to dq axis

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \sin \delta & -\cos \delta \\ \cos \delta & \sin \delta \end{bmatrix} \begin{bmatrix} V_r \\ V_i \end{bmatrix}$$

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} 0.886987 & -0.461794 \\ 0.461794 & 0.886987 \end{bmatrix} \begin{bmatrix} 0.947150 \\ 0.310351 \end{bmatrix} = \begin{bmatrix} 0.6968 \\ 0.7127 \end{bmatrix}$$



The screenshot shows a software interface with three sections:

- Terminal Voltage and Current:** A table with two columns: "Magnitude (pu)" and "Angle (deg)". The "Voltage" row shows a magnitude of 0.9967 and an angle of 18.1423.
- Internal Angle:** A field for "Internal Angle Delta (deg)" with a value of 62.4971.
- Direct and Quadrature Axis Values:** A table with two columns: "Direct Axis" and "Quadrature Axis". The "Terminal Voltage (pu)" row shows values of 0.6968 and 0.7127.

Color-coded boxes highlight the values used in the calculations: red for 0.9967, blue for 18.1423, green for 62.4971, and yellow for the final dq-axis voltage components 0.6968 and 0.7127.

Seeing the Model States



- On Stability Tab, then Terminal and State, then Terminal Values

δ
 ω
 E'_q
 ψ'_d
 ψ''_q
 E'_d

Generator MVA Base: 1559.00 Unit Type: CA (Combined Cycle Steam Part) ▾

Power and Voltage Control Costs OPF Faults Owners, Area, etc. Custom **Stability**

Machine Models Exciters Governors Stabilizers Other Models Step-up Transformer **Terminal and State**

Simulation Time (seconds): Initialized at 0.000000 Save Two Bus Equivalent

PU values shown/entered using device base of 1559.0 MVA ▾

Bus/Setpoint Values Terminal Values **Machine States** Exciter States Governor States Stabilizer States

Active Model Type: GENROU

Angle	1.0908	Derivative of Angle	0.00000
Speed w	0.0000	Derivative of Speed w	0.00000
Eqp	0.9978	Derivative of Eqp	0.00000
PsiDp	0.8830	Derivative of PsiDp	0.00000
PsiQpp	0.5926	Derivative of PsiQpp	0.00000
Edp	0.4254	Derivative of Edp	0.00000

**$\delta = 1.0908$ radians
 $= 62.4971$ degrees**

Model Simplifications



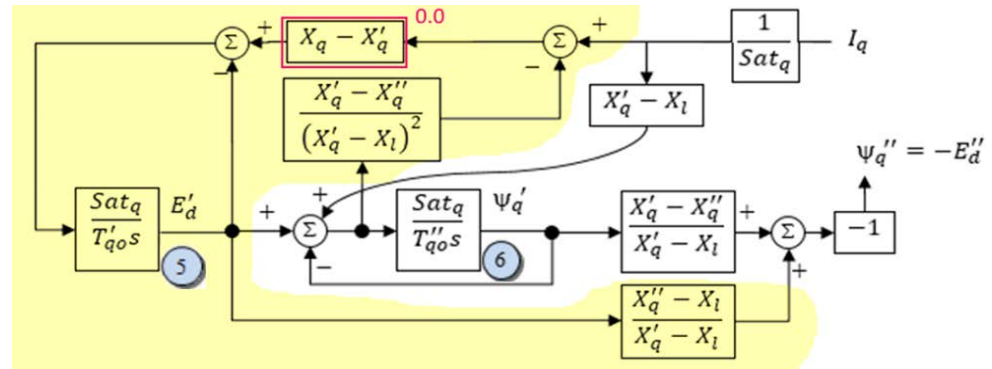
- Newer models (GENTPF/GENTPJ, GENQEC) have particular input parameter combinations to indicate model simplifications
- Salient Pole Machine with a single amortisseur winding
 - $X_q = X'_q$
 - $T'_{qo} = 0$
- Salient Pole Machine without any amortisseur windings
 - $X_q = X'_q = X''_q$
 - $X'_d = X''_d$
 - $T'_{qo} = T''_{do} = T''_{qo} = 0$
 - $T'_{do} > 0$

Simplification for GENQEC for Salient Pole with 1 amortisseur

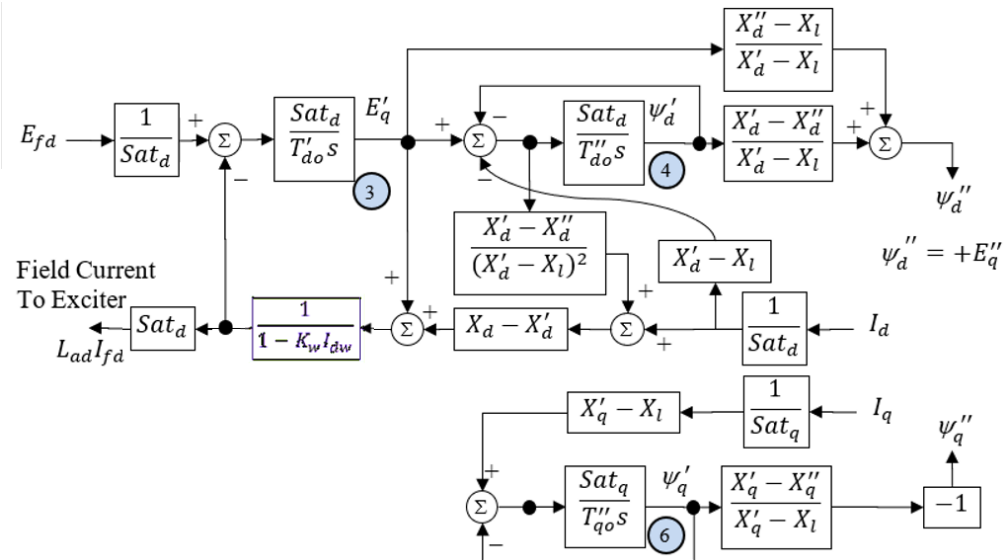


$$X_q = X'_q \text{ and } T'_{qo} = 0,$$

- q-axis is simplified to



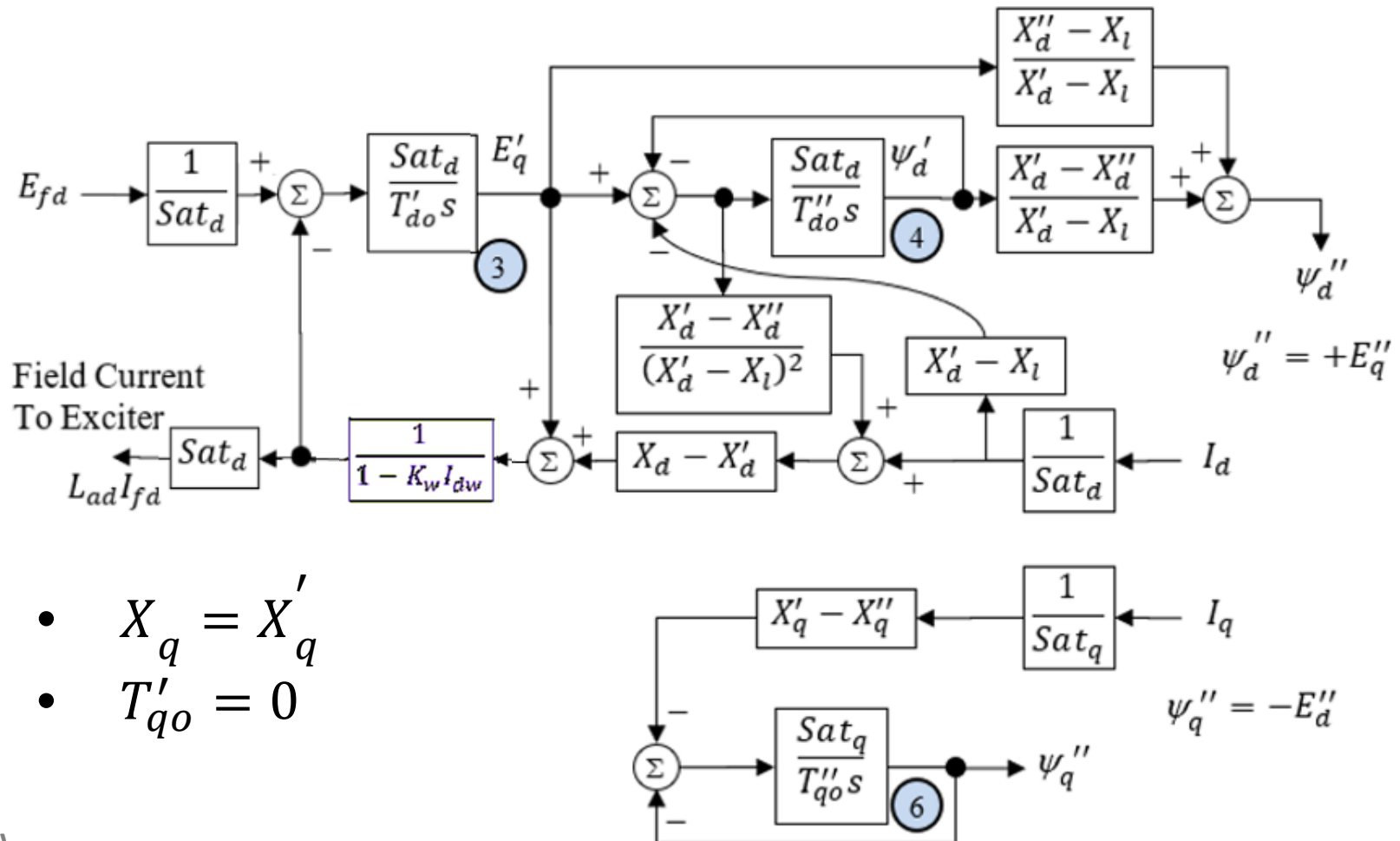
- This becomes



GENQEC simplified

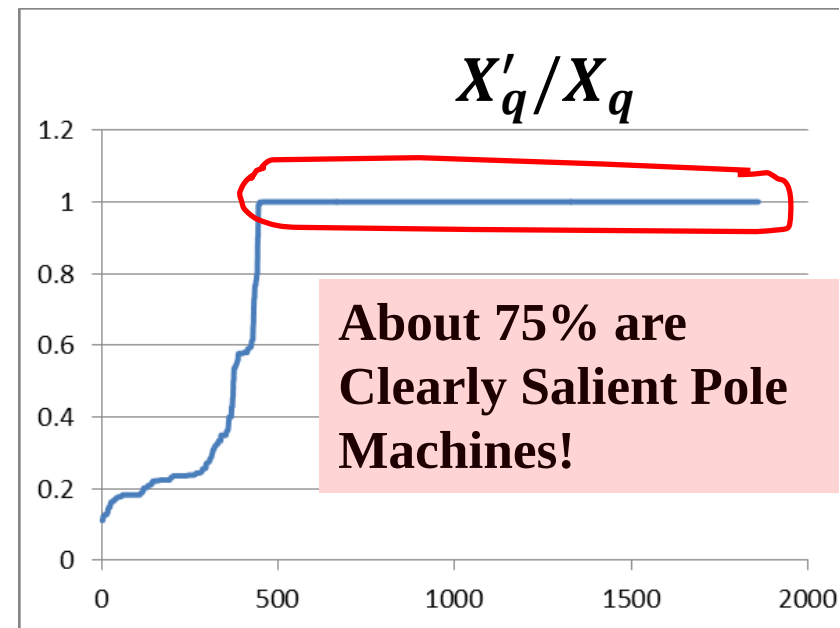
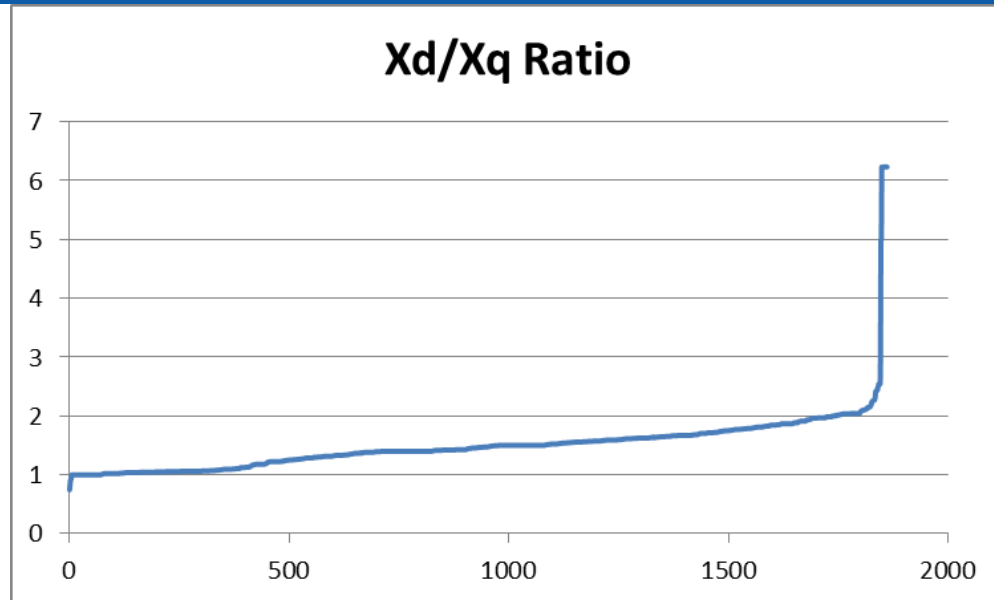


- Salient Pole with 1 amortisseur

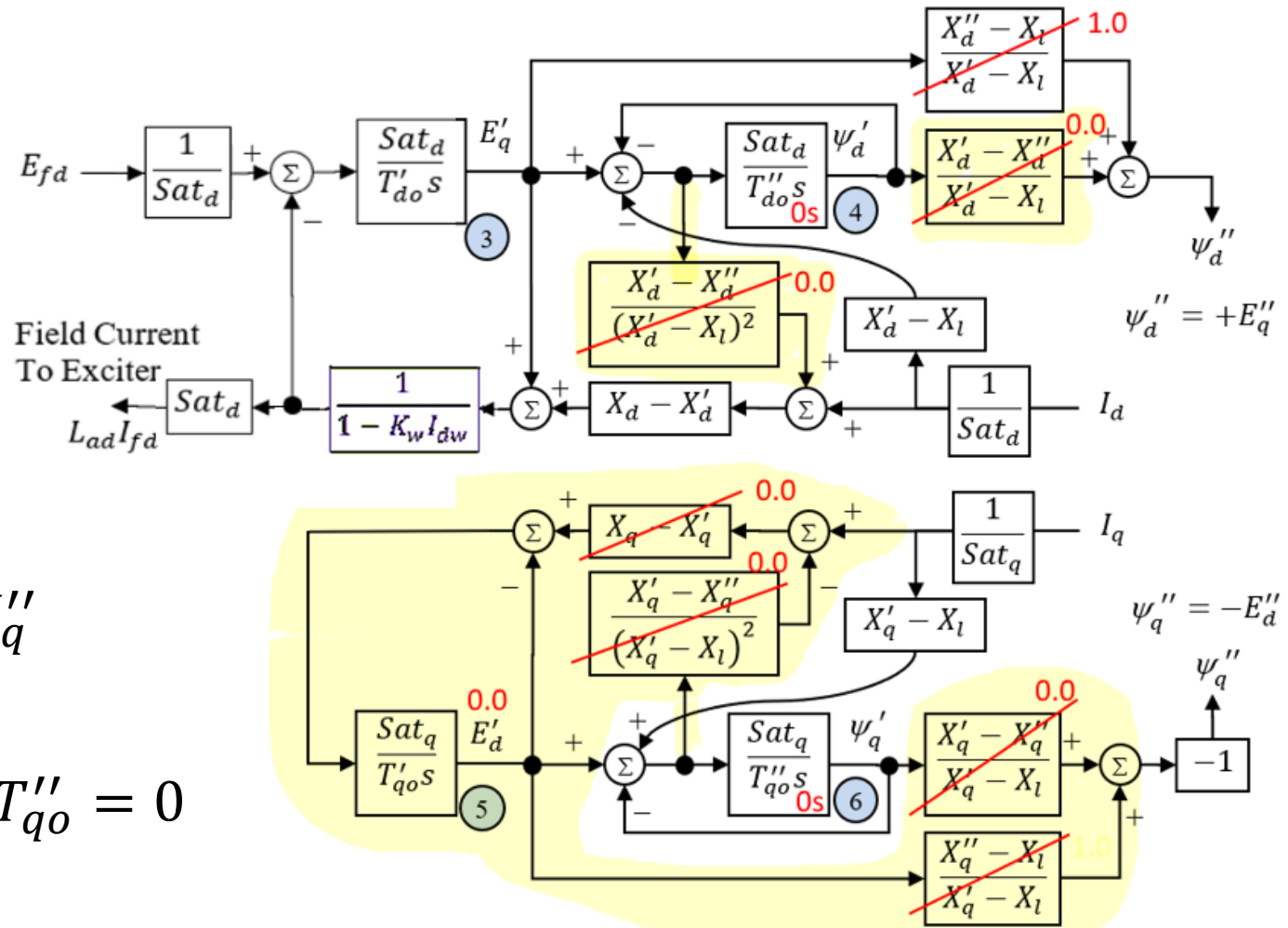


- $X_q = X'_q$
- $T'_{qo} = 0$

Reminder: X_d and X_q have physical meaning: Example WECC



Simplification for GENQEC for Salient Pole with no amortisseur

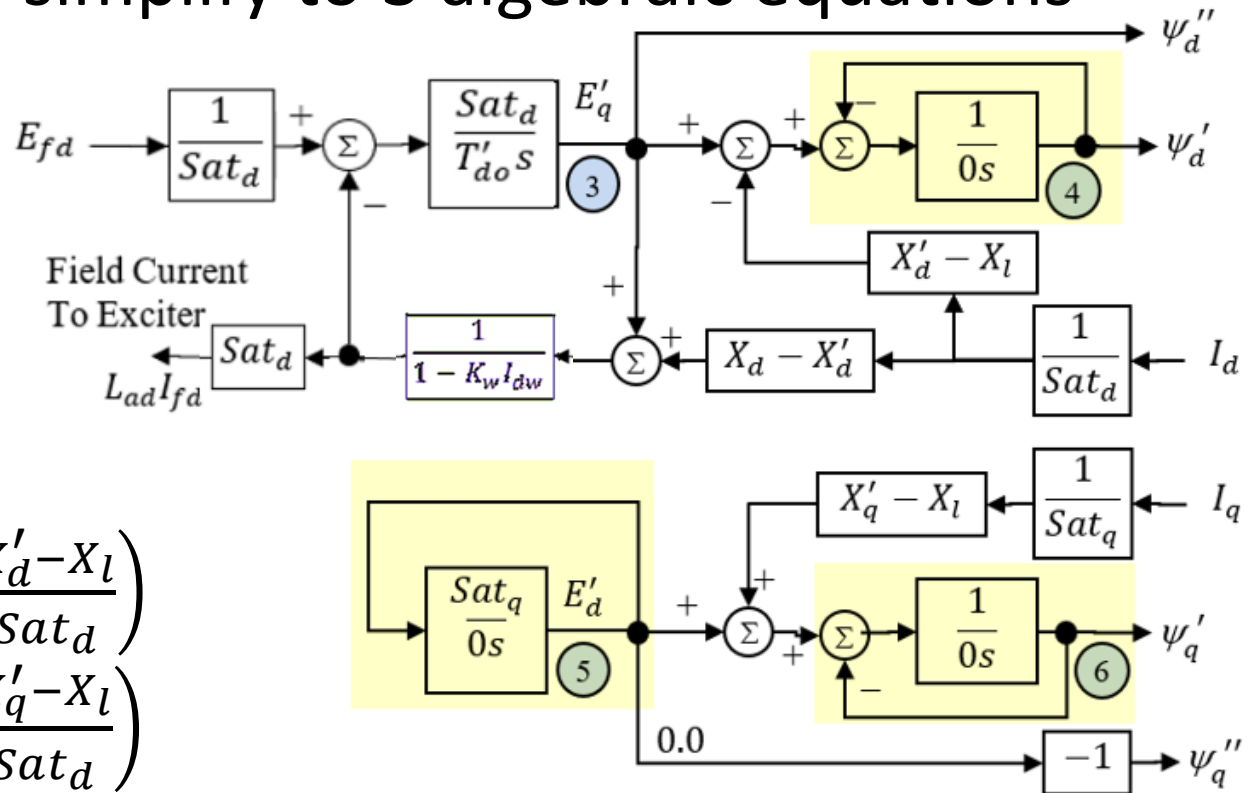


- $X_q = X'_q = X''_q$
- $X'_d = X''_d$
- $T'_{qo} = T''_{do} = T''_{qo} = 0$
- $T'_{do} > 0$

Simplification for GENQEC for Salient Pole with 0 amortisseur



- This becomes the following. The yellow highlighted blocks are infinitely fast delay blocks which simplify to 3 algebraic equations



- $E'_d = 0$
- $\psi'_d = E'_q - I_d \left(\frac{X'_d - X_l}{Sat_d} \right)$
- $\psi'_q = E'_d + I_q \left(\frac{X'_q - X_l}{Sat_d} \right)$

