

Linear Analysis Techniques in PowerWorld Simulator



An overview of the underlying mathematics of the
power flow and linearized analysis techniques



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AC Power Flow Equations



- Full AC Power Flow Equations

$$P_k = 0 = V_k^2 g_{kk} + V_k \sum_{\substack{m=1 \\ m \neq k}}^N (V_m [g_{km} \cos(\delta_k - \delta_m) + b_{km} \sin(\delta_k - \delta_m)]) - P_{Gk} + P_{Lk}$$

$$Q_k = 0 = -V_k^2 b_{kk} + V_k \sum_{\substack{m=1 \\ m \neq k}}^N (V_m [g_{km} \sin(\delta_k - \delta_m) - b_{km} \cos(\delta_k - \delta_m)]) - Q_{Gk} + Q_{Lk}$$

- Solution requires iteration of equations

$$\begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathbf{P}}{\partial \delta} & \frac{\partial \mathbf{P}}{\partial V} \\ \frac{\partial \mathbf{Q}}{\partial \delta} & \frac{\partial \mathbf{Q}}{\partial V} \end{bmatrix}^{-1} \begin{bmatrix} \Delta \mathbf{P} \\ \Delta \mathbf{Q} \end{bmatrix} = [\mathbf{J}]^{-1} \begin{bmatrix} \Delta \mathbf{P} \\ \Delta \mathbf{Q} \end{bmatrix}$$

- Note: the large matrix (J) is called the Jacobian

Full AC Derivatives



- Real Power derivative equations are

$$\frac{\partial P_k}{\partial \delta_m} = V_k V_m [-g_{km} \cos(\delta_k - \delta_m) - b_{km} \sin(\delta_k - \delta_m)]$$

$$\frac{\partial P_k}{\partial V_m} = V_k [g_{km} \sin(\delta_k - \delta_m) - b_{km} \cos(\delta_k - \delta_m)]$$

$$\frac{\partial P_k}{\partial \delta_k} = V_k \sum_{\substack{m=1 \\ m \neq k}}^N [V_m [-g_{km} \sin(\delta_k - \delta_m) + b_{km} \cos(\delta_k - \delta_m)]]$$

$$\frac{\partial P_k}{\partial V_k} = 2V_k g_{kk} + \sum_{\substack{m=1 \\ m \neq k}}^N [V_m [g_{km} \cos(\delta_k - \delta_m) + b_{km} \sin(\delta_k - \delta_m)]]$$

- Reactive Power derivative equations are

$$\frac{\partial Q_k}{\partial \delta_m} = V_k V_m [-g_{km} \cos(\delta_k - \delta_m) - b_{km} \sin(\delta_k - \delta_m)]$$

$$\frac{\partial Q_k}{\partial V_m} = V_k [g_{km} \sin(\delta_k - \delta_m) - b_{km} \cos(\delta_k - \delta_m)]$$

$$\frac{\partial Q_k}{\partial \delta_k} = V_k \sum_{\substack{m=1 \\ m \neq k}}^N [V_m [g_{km} \cos(\delta_k - \delta_m) + b_{km} \sin(\delta_k - \delta_m)]]$$

$$\frac{\partial Q_k}{\partial V_k} = -2V_k b_{kk} + \sum_{\substack{m=1 \\ m \neq k}}^N [V_m [g_{km} \sin(\delta_k - \delta_m) - b_{km} \cos(\delta_k - \delta_m)]]$$

Decoupled Power Flow Equations



- Make the following assumptions
- Derivatives simplify to

$$\begin{aligned} \delta_k - \delta_m &\approx 0 & \sin(\delta_k - \delta_m) &\approx 0 \\ V_k &\approx 1 & g_{km} &\approx 0 & \cos(\delta_k - \delta_m) &\approx 1 \end{aligned}$$

$$\frac{\partial P_k}{\partial \delta_k} = \sum_{\substack{m=1 \\ m \neq k}}^N b_{km}$$

$$\frac{\partial Q_k}{\partial V_k} = -2b_{kk} + \sum_{\substack{m=1 \\ m \neq k}}^N (-b_{km})$$

$$\begin{aligned} \frac{\partial P_k}{\partial \delta_m} &= -b_{km} \\ \frac{\partial Q_k}{\partial V_m} &= -b_{km} \end{aligned}$$

$$\begin{aligned} \frac{\partial P_k}{\partial V_k} &= 0 & \frac{\partial P_k}{\partial V_m} &= 0 \\ \frac{\partial Q_k}{\partial \delta_k} &= 0 & \frac{\partial Q_k}{\partial \delta_m} &= 0 \end{aligned}$$

- Note: If assumption is instead $r_{km} \approx 0$ then replace all b_{km} with $1/x_{km}$
- Option in Simulator Options under Power Flow Solution, DC Options sub-tab

DC Power Flow Line Model

- ☒ Ignore Line Series Resistance (r)
- ☐ Ignore Line Series Conductance (g)

Other Typical Assumptions about Transformers



- Ignore Transformer Impedance Correction Tables
 - If we do not ignore, then as tap or phase changes the series branch impedances change
- Ignore Phase Shift Angle Effects
 - If we do not ignore, then as the phase angle changes the series impedance seen between the buses will vary
- Both of these are normally done because if they are not ignored, then the solution matrices become a function of the system state (i.e. impedance will vary with tap or phase)
 - This defeats the purposed of using the Decoupled equations.

Recommendation:
Leave these checked

A screenshot of a software dialog box titled "DC Power Flow Model". It contains a note about settings affecting Lossless DC sensitivity calculations. Below the note, there are two sections: "DC Power Flow Line Model" with radio buttons for "Ignore Line Series Resistance (r)" (selected) and "Ignore Line Series Conductance (g)", and a section with two checked checkboxes: "Ignore Transformer Impedance Correction Tables" and "Ignore Phase Shift Angle Effects". Two red arrows point from the text "Leave these checked" to these two checkboxes.

DC Power Flow Model

Note: These settings also affect the Lossless DC sensitivity calculations such as on the PTDF and TLR/GSF dialogs

DC Power Flow Line Model

☒ Ignore Line Series Resistance (r)

☐ Ignore Line Series Conductance (g)

☒ Ignore Transformer Impedance Correction Tables

☒ Ignore Phase Shift Angle Effects

B' and B'' Matrices



- Define $\frac{\partial \mathbf{Q}}{\partial \mathbf{V}} = \mathbf{B}''$ $\frac{\partial \mathbf{P}}{\partial \delta} = \mathbf{B}'$

- Now Iterate the “decoupled” equations

$$\Delta \delta = [\mathbf{B}']^{-1} \Delta \mathbf{P}$$
$$\Delta \mathbf{V} = [\mathbf{B}'']^{-1} \Delta \mathbf{Q}$$

- What are B' and B''?
 - B' is the imaginary part of the Y-Bus with all the “shunt terms” removed
 - B'' is the imaginary part of the Y-Bus with all the “shunt terms” double counted

“DC Power Flow”



- The “DC Power Flow” equations are simply the real part of the decoupled power flow equations
 - Voltages and reactive power are ignored
 - Only angles and real power are solved for by iterating

$$\Delta\delta = [\mathbf{B}']^{-1} \Delta\mathbf{P}$$

Bus Voltage and Angle Sensitivities to a Transfer



- Power flow was solved by iterating
$$\begin{bmatrix} \Delta \delta \\ \Delta \mathbf{V} \end{bmatrix} = [\mathbf{J}]^{-1} \begin{bmatrix} \Delta \mathbf{P} \\ \Delta \mathbf{Q} \end{bmatrix}$$
- Model the transfer as a change in the injections $\Delta \mathbf{P}$
 - Buyer: $\Delta \mathbf{T}_B = [0 \quad 0 \quad PF_{Bf} \quad 0 \quad PF_{Bg} \quad 0]^T \quad \sum_{h=1}^N PF_{Bh} = 1$
 - Seller: $\Delta \mathbf{T}_S = [0 \quad PF_{Sx} \quad 0 \quad PF_{Sy} \quad 0 \quad 0]^T \quad \sum_{z=1}^N PF_{Sz} = 1$
- Then solve for the voltage and angle sensitivities by solving

$$\begin{bmatrix} \Delta \delta_S \\ \Delta \mathbf{V}_S \end{bmatrix} = [\mathbf{J}]^{-1} \begin{bmatrix} \Delta \mathbf{T}_S \\ \mathbf{0} \end{bmatrix} \quad \begin{bmatrix} \Delta \delta_B \\ \Delta \mathbf{V}_B \end{bmatrix} = [\mathbf{J}]^{-1} \begin{bmatrix} \Delta \mathbf{T}_B \\ \mathbf{0} \end{bmatrix}$$
- These are the sensitivities of the Buyer and Seller “sending power to the slack bus”

What about Losses?



- If we assume the total sensitivity to the transfer is the seller minus the buyer sensitivity, then $\Delta\delta = \Delta\delta_S - \Delta\delta_B$ and $\Delta\mathbf{V} = \Delta\mathbf{V}_S - \Delta\mathbf{V}_B$
- This makes assumption that ALL the change in losses shows up at the slack bus.

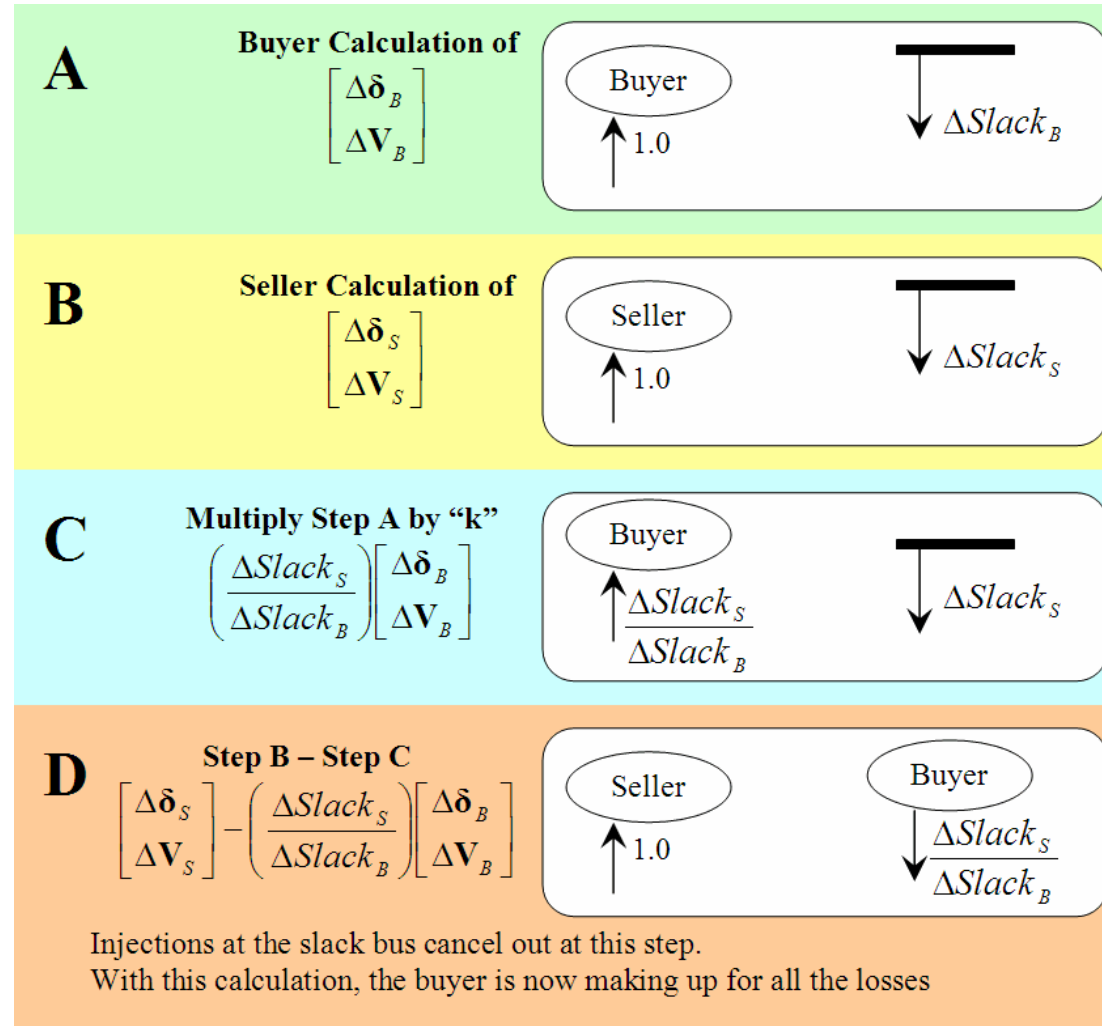
- Simulator assigns the change to the BUYER by defining

$$k = \frac{\Delta\text{Slack}_S}{\Delta\text{Slack}_B} = \frac{\text{Change in slack bus generation for seller sending power to slack}}{\text{Change in slack bus generation for buyer sending power to slack}}$$

- Then

$$\begin{bmatrix} \Delta\delta \\ \Delta\mathbf{V} \end{bmatrix} = \begin{bmatrix} \Delta\delta_S \\ \Delta\mathbf{V}_S \end{bmatrix} - k \begin{bmatrix} \Delta\delta_B \\ \Delta\mathbf{V}_B \end{bmatrix}$$

Why does “k” assign the losses to the Buyer?



Lossless DC Voltage and Angle Sensitivities



- Use the DC Power Flow Equations

$$\Delta\delta = [\mathbf{B}']^{-1} \Delta\mathbf{P}$$

- Then determine angle sensitivities

$$\Delta\delta_S = [\mathbf{B}']^{-1} \Delta\mathbf{T}_S \quad \Delta\delta_B = [\mathbf{B}']^{-1} \Delta\mathbf{T}_B$$

- The DC Power Flow ignores losses, thus

$$\Delta\delta = \Delta\delta_S - \Delta\delta_B$$

Lossless DC Sensitivities with Phase Shifters Included



- DC Power Flow equations $[\mathbf{B}']\Delta\delta = \Delta\mathbf{P}$
- Augmented to include an equation that describes the change in flow on a phase-shifter controlled branch as being zero.

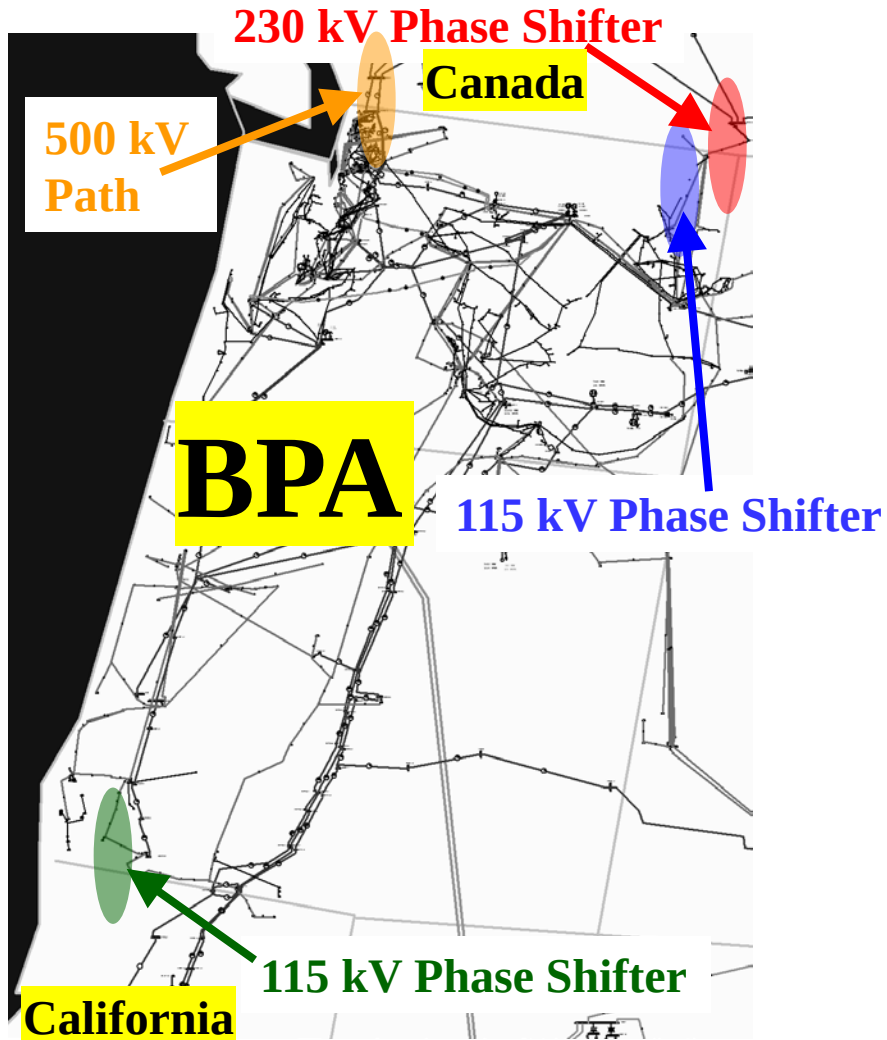
$$\text{Line Flow Change} = \mathbf{B}_{\delta}\Delta\delta + \mathbf{B}_{\alpha}\Delta\alpha = \mathbf{0}$$

- Thus instead of DC power flow equations we use

$$\begin{bmatrix} \Delta\delta \\ \Delta\alpha \end{bmatrix} = \begin{bmatrix} \mathbf{B}' & \mathbf{0} \\ \mathbf{B}_{\delta} & \mathbf{B}_{\alpha} \end{bmatrix}^{-1} \begin{bmatrix} \Delta\mathbf{P} \\ \mathbf{0} \end{bmatrix}$$

- Otherwise process is the same.

Lossless DC with Phase Shifters



- Phase Shifters are often on lower voltage paths (230 kV or less) with relatively small limits
 - They are put there in order to manage/prevent the flow on a path that would commonly see overloads
 - Thus, they constantly show up as “overloaded” when using linear analysis if they are not accounted for
- Example: Border of Canada with Northwestern United States
 - PTDFs between Canada and US without Phase-Shifters
 - 85% on 500 kV Path
 - 15% on Eastern Path
 - PTDF With Phase-Shifters
 - 100% goes on 500 kV Path
 - 0% on Eastern Path
 - This better reflects real system

Power Transfer Distribution Factors (PTDFs)



- PTDF: measures the sensitivity of line MW flows to a MW transfer.
 - Line flows are simply a function of the voltages and angles at its terminal buses
 - Thus, the PTDF is simply a function of these voltage and angle sensitivities.
 - This is the “Chain Rule” from Calculus
- P_{km} is the flow from bus k to bus m

$$PTDF = \Delta P_{km} = \left[\frac{\partial P_{km}}{\partial V_k} \right] \Delta V_k + \left[\frac{\partial P_{km}}{\partial V_m} \right] \Delta V_m + \left[\frac{\partial P_{km}}{\partial \delta_k} \right] \Delta \delta_k + \left[\frac{\partial P_{km}}{\partial \delta_m} \right] \Delta \delta_m$$

Voltage and Angle Sensitivities that were just determined

P_{km} Derivative Calculations



- Full AC equations

$$\left[\frac{\partial P_{km}}{\partial \delta_m} \right] = V_k V_m \left[g_{km} \sin(\delta_k - \delta_m) - b_{km} \cos(\delta_k - \delta_m) \right]$$

$$\left[\frac{\partial P_{km}}{\partial \delta_k} \right] = V_k V_m \left[-g_{km} \sin(\delta_k - \delta_m) + b_{km} \cos(\delta_k - \delta_m) \right]$$

$$\left[\frac{\partial P_{km}}{\partial V_m} \right] = V_k \left[g_{km} \cos(\delta_k - \delta_m) + b_{km} \sin(\delta_k - \delta_m) \right]$$

$$\left[\frac{\partial P_{km}}{\partial V_k} \right] = 2V_k g_{kk} + V_m \left[g_{km} \cos(\delta_k - \delta_m) + b_{km} \sin(\delta_k - \delta_m) \right]$$

- Lossless DC Approximations yields

$$\left[\frac{\partial P_{km}}{\partial \delta_m} \right] = -b_{km}$$

$$\left[\frac{\partial P_{km}}{\partial \delta_k} \right] = b_{km}$$

$$\left[\frac{\partial P_{km}}{\partial V_k} \right] = 0$$

$$\left[\frac{\partial P_{km}}{\partial V_m} \right] = 0$$

What do Flow Sensitivities (PTDFs, GSFs, TLRs, ...) give us?



- Give the ability to model a change in power injection without actually doing a new power flow solution
 - Transfer of power between two places
 - Generator outage
 - Load outage
- Still can't model a line outage or line closure yet, but that is what LODFs will give us

Line Outage Distribution Factors (LODFs)

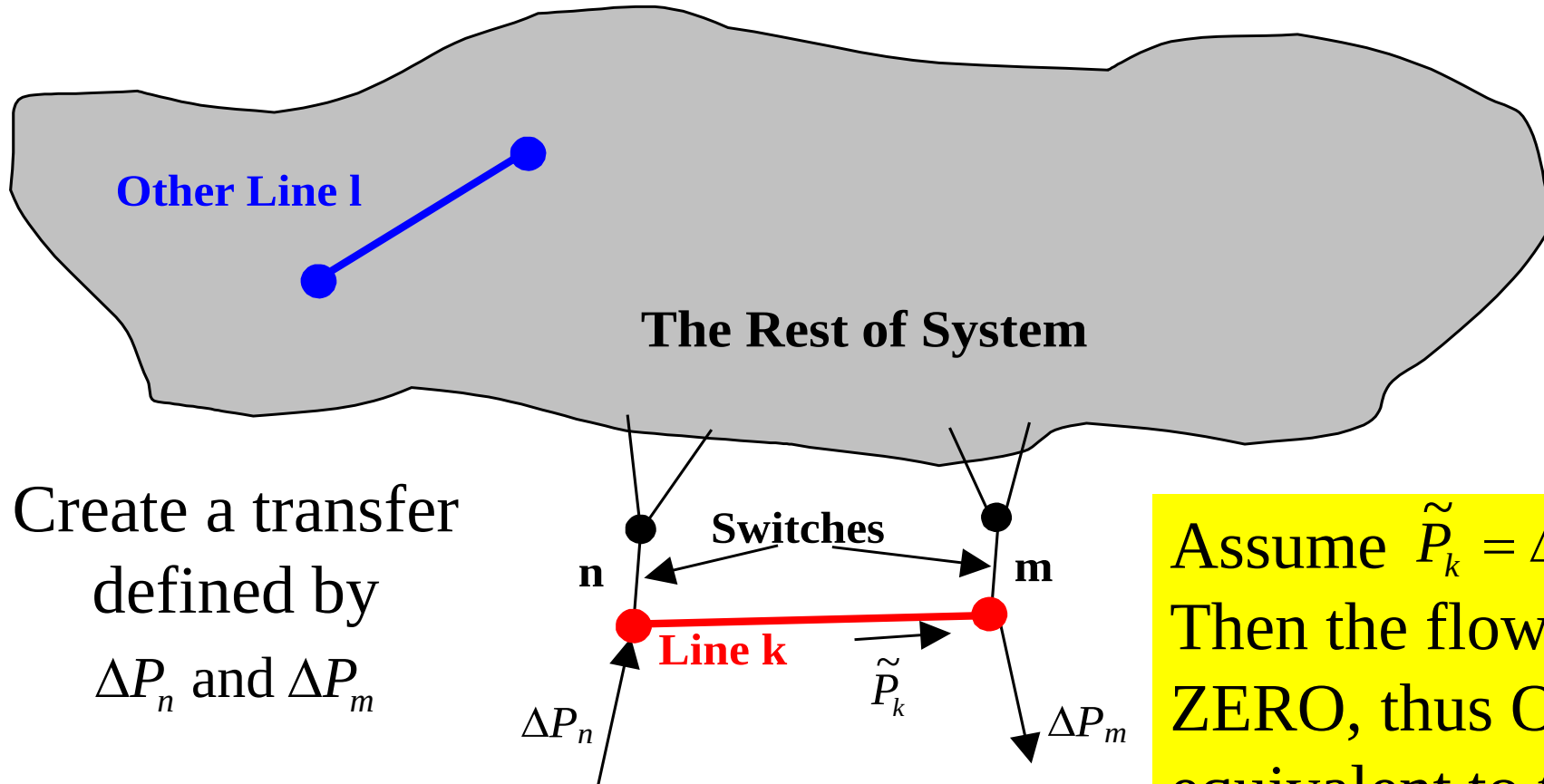


- $LODF_{l,k}$: percent of the pre-outage flow on Line K will show up on Line L after the outage of Line K

$$LODF_{l,k} = \frac{\Delta P_{l,k}}{P_k}$$

- Linear impact of an outage is determined by modeling the outage as a “transfer” between the terminals of the line

Modeling an LODF as a Transfer



Create a transfer
defined by
 ΔP_n and ΔP_m

Assume $\tilde{P}_k = \Delta P_n = \Delta P_m$
Then the flow on the Switches is
ZERO, thus Opening Line K is
equivalent to the “transfer”

Modeling an LODF as a Transfer



- Thus, setting up a transfer of $\tilde{P}_k$ MW from Bus n to Bus m is equivalent to outaging the transmission line
- Let's assume we know $\tilde{P}_k$; then we can calculate the values relevant to the LODF.

Calculation of LODF



- Estimate of post-outage flow on Line L

$$\Delta P_{l,k} = PTDF_l * \tilde{P}_k$$

- Estimate of flow on Line L after transfer

$$\tilde{P}_k = P_k + PTDF_k * \tilde{P}_k \longrightarrow \tilde{P}_k = \frac{P_k}{1 - PTDF_k}$$

- Thus we can write

$$LODF_{l,k} = \frac{\Delta P_{l,k}}{P_k} = \frac{PTDF_l * \tilde{P}_k}{P_k} = \frac{PTDF_l * \left(\frac{P_k}{1 - PTDF_k} \right)}{P_k}$$
$$\longrightarrow LODF_{l,k} = \frac{PTDF_l}{1 - PTDF_k}$$

- We have a simple function of PTDF values

Line Closure Distribution Factors (LCDFs)

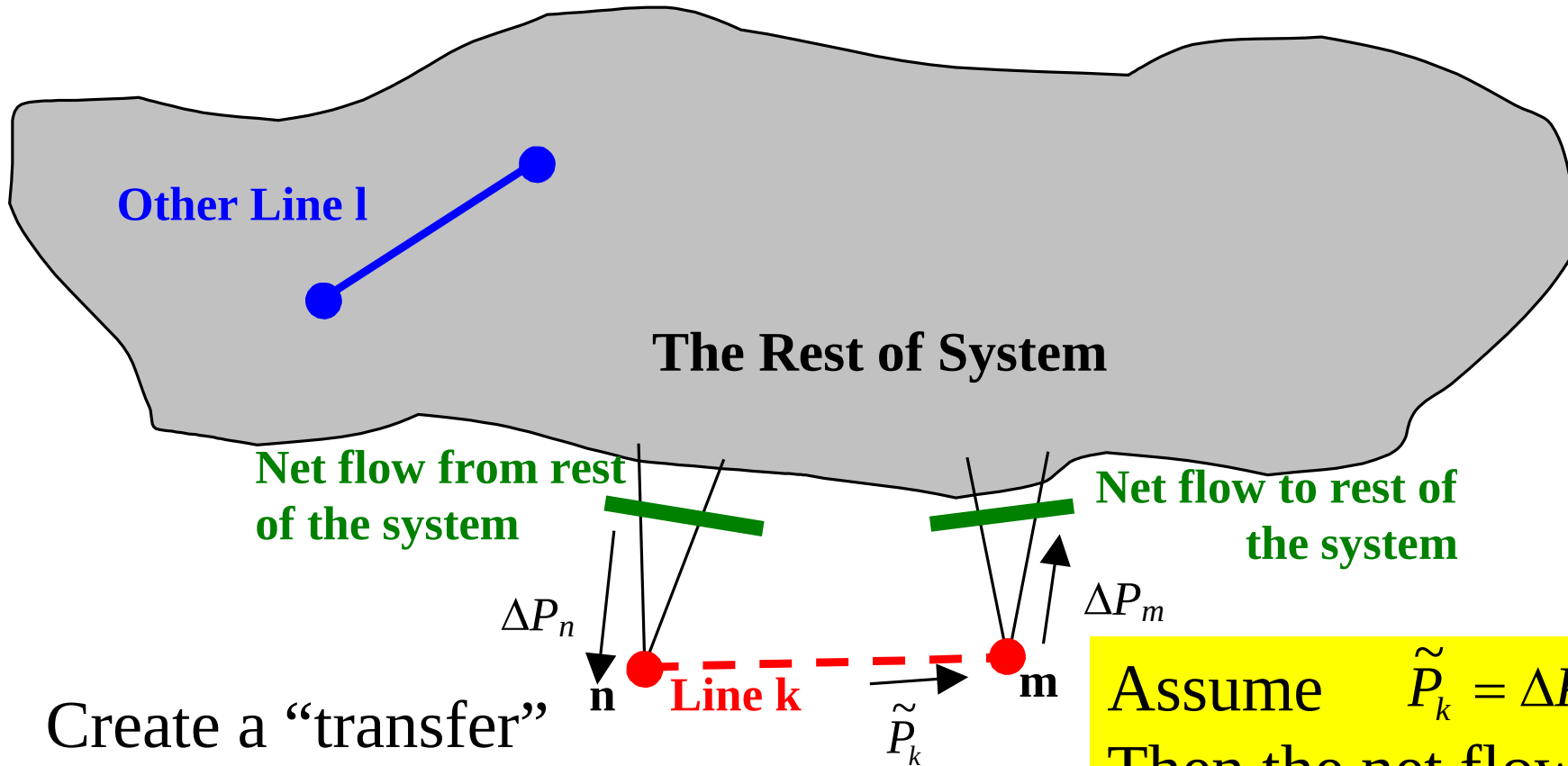


- $LCDF_{l,k}$: percent of the post-closure flow on Line K will show up on Line L after the closure of Line K

$$LCDF_{l,k} = \frac{\Delta P_{l,k}}{\tilde{P}_k}$$

- Linear impact of an closure is determined by modeling the closure as a “transfer” between the terminals of the line

Modeling the LCDF as a Transfer



Create a “transfer”
defined by
 ΔP_n and ΔP_m

Assume $\tilde{P}_k = \Delta P_n = \Delta P_m$
Then the net flow to and from the rest of
the system are both zero, thus closing
line k is equivalent the “transfer”

Modeling an LCDF as a Transfer



- Thus, setting up a transfer of $-\tilde{P}_k$ MW from Bus n to Bus m is linearly equivalent to outaging the transmission line
- Let's assume we know $-\tilde{P}_k$; then we can calculate the values relevant to the LODF.
- Note: The negative sign is used so that the notation is consistent with LODF “transfer”

Calculation of LCDF



- Estimate of post-closure flow on Line L

$$\Delta P_{l,k} = -PTDF_l * \tilde{P}_k$$

- Thus we can write

$$LCDF_{l,k} = \frac{\Delta P_{l,k}}{\tilde{P}_k} = \frac{-PTDF_l * \tilde{P}_k}{\tilde{P}_k} = -PTDF_l$$

$$\longrightarrow LCDF_{l,k} = -PTDF_l$$

- Thus the LCDF is exactly equal to the PTDF for a transfer between the terminals of the line

LODF and LCDF

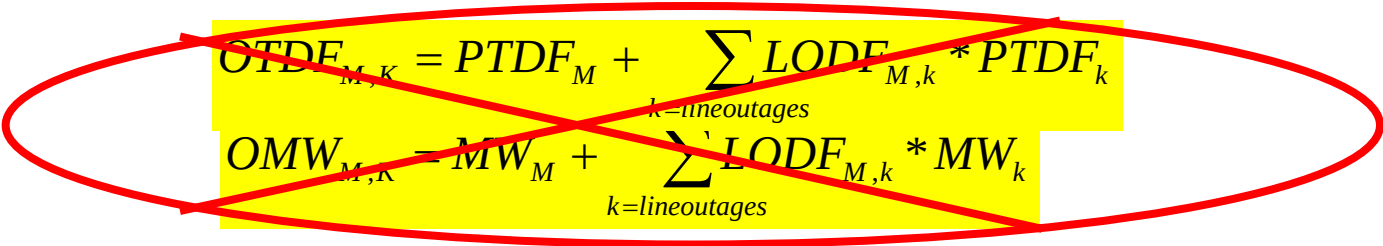


- LODF (LCDF) – Gives the ability to model a single line outage (closure) event
 - OTDF – incremental impact on a particular branch while also considering a single line outage
 - OMW – estimate of the flow on a particular branch after a single line outage

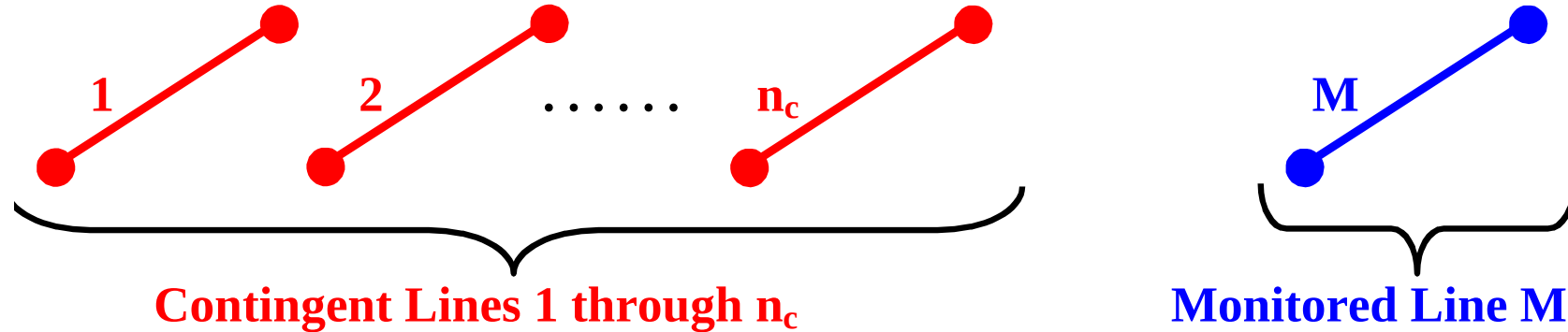
$$OTDF_{M,1} = PTDF_M + LODF_{M,1} * PTDF_1$$

$$OMW_{M,1} = MW_M + LODF_{M,1} * MW_1$$

- Still can't model multiple line outages simultaneously
 - Can not just sum because the lines that are being outaged also interact with each other


$$OTDF_{M,K} = PTDF_M + \sum_{k=lineoutages} LODF_{M,k} * PTDF_k$$
$$OMW_{M,K} = MW_M + \sum_{k=lineoutages} LODF_{M,k} * MW_k$$

Linear Impact of a Contingency with Multiple Outages



- Outage Transfer Distribution Factors (OTDFs)
 - The percent of a transfer that will flow on a branch M after the contingency occurs
- Outage Flows (OMWs)
 - The estimated flow on a branch M after the contingency occurs

OTDFs and OMWs



- Single Contingency

$$OTDF_{M,1} = PTDF_M + LODF_{M,1} * PTDF_1$$

$$OMW_{M,1} = MW_M + LODF_{M,1} * MW_1$$

- Multiple Contingencies

$$OTDF_{M,C} = PTDF_M + \sum_{K=1}^{n_C} [LODF_{MK} * NetPTDF_K]$$

$$OMW_{M,C} = MW_M + \sum_{K=1}^{n_C} [LODF_{MK} * NetMW_K]$$

- What are $NetPTDF_K$ and $NetMW_K$?

Determining $NetPTDF_K$ and $NetMW_K$



- Each $NetPTDF_K$ is a function of all the other $NetPTDFs$ because the change in status of a line affects all other lines.
- Assume we know all $NetPTDFs$ except for $NetPTDF_1$. Then we can write:

$$\begin{aligned} NetPTDF_1 &= PTDF_1 + LODF_{12} NetPTDF_2 + \dots + LODF_{1n_c} NetPTDF_{n_c} \\ &= PTDF_1 + \sum_{K=2}^{n_c} [LODF_{1K} NetPTDF_K] \end{aligned}$$

- In general, for each Contingent Line N, write

$$NetPTDF_N - \sum_{\substack{K=1 \\ K \neq N}}^{n_c} [LODF_{NK} NetPTDF_K] = PTDF_N$$

Determining $NetPTDF_K$ and $NetMW_K$



- Thus we have a set of n_c equations and n_c unknowns (n_c = number of contingent lines)

Known Values

$$\begin{bmatrix} 1 & -LODF_{12} & -LODF_{13} & \cdots & -LODF_{1n_c} \\ -LODF_{21} & 1 & -LODF_{23} & \cdots & -LODF_{2n_c} \\ -LODF_{31} & -LODF_{32} & 1 & \cdots & -LODF_{3n_c} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -LODF_{n_c1} & -LODF_{n_c2} & -LODF_{n_c3} & \cdots & 1 \end{bmatrix} \begin{bmatrix} NetPTDF_1 \\ NetPTDF_2 \\ NetPTDF_3 \\ \vdots \\ NetPTDF_{n_c} \end{bmatrix} = \begin{bmatrix} PTDF_1 \\ PTDF_2 \\ PTDF_3 \\ \vdots \\ PTDF_{n_c} \end{bmatrix}$$

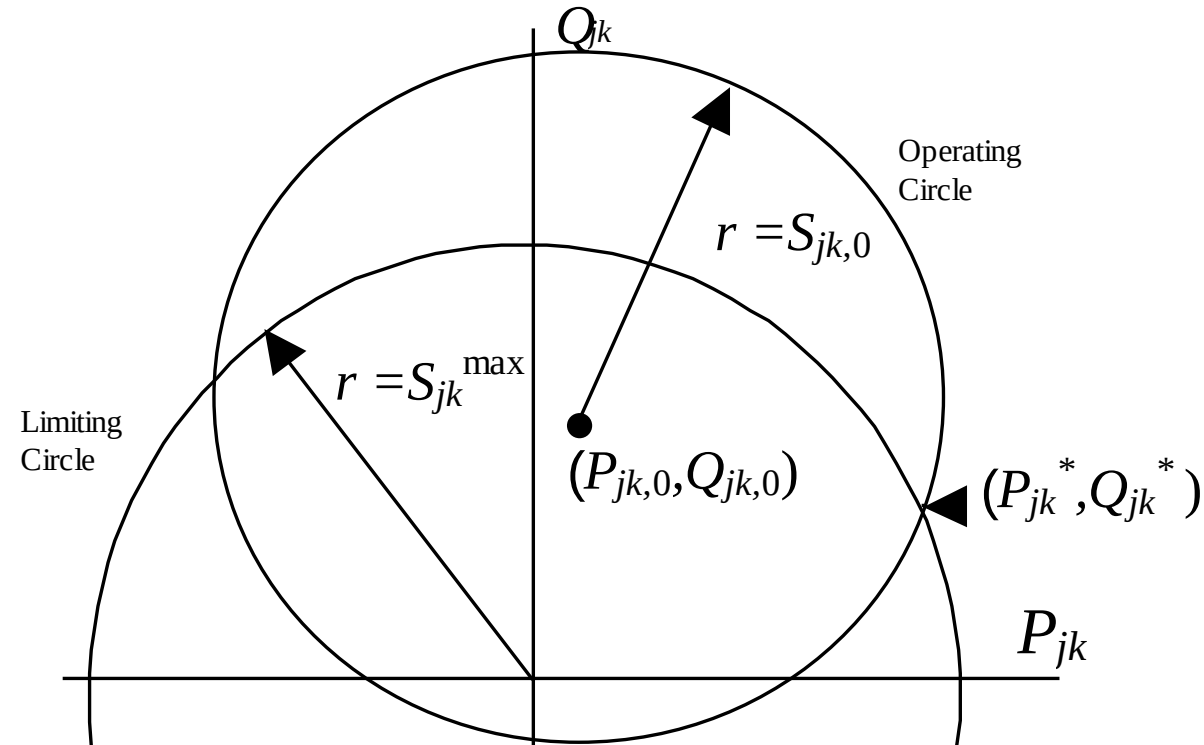
- Thus $NetPTDF_c = [LODF_{cc}]^{-1} PTDF_c$
- Same type of derivation shows $NetMW_c = [LODF_{cc}]^{-1} MW_c$

Operating and Limiting Circles



- Operating circle defines a circle of valid MW and Mvar values for a transmission line as a transfer takes place across the system
- Limiting circle has a radius equal to the MVA limit of the line
- The operating circle is utilized when using one of the DC methods and modeling reactive power by assuming constant voltage magnitudes
- Contingency analysis looks up the Mvar value corresponding to the calculated MW value
- ATC analysis and DC power flow find the intersection of the operating circle and the limiting circle to assign adjusted limits to lines

Operating and Limiting Circles



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